University of Saskatchewan
Department of Electrical and Computer Engineering

Final Examination of **EE362: Digital Signal Processing**
December 16, 2020
3 hours

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10 points

1. Consider the discrete-time system shown in Fig. 1. For each of the following input signals $x[n]$, indicate (and justify) whether the output $x_r[n] = x[n]$.

   - $x[n] = \cos(\pi n/4)$
   - $x[n] = \cos(\pi n/2)$
   - $x[n] = (\sin(\pi n/8))^2 / (\pi n)^2$, **hint:** Use the modulation property of the Fourier transform to find $X(e^{jw})$.

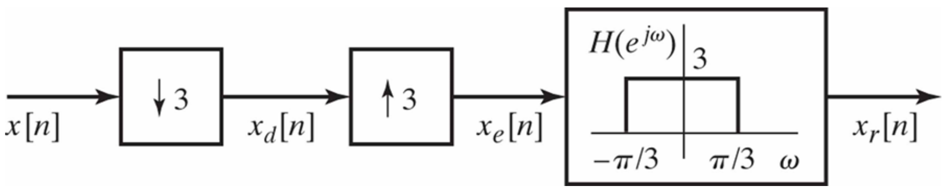

Figure 1: Discrete-time system.

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4 points

2. Show that the function $x[n] = \alpha^n$, where α is a complex constant, is an eigenfunction of an LTI discrete-time system. Is $y[n] = \alpha^n u[n]$, where $u[n]$ is the unit step sequence, also an eigenfunction of an LTI discrete-time system?

6 points

3. Determine a closed-form expression for the frequency response $H(e^{jw})$ of the LTI discrete-time system characterized by the following impulse response

$$h[n] = \delta[n] - \alpha\delta[n-5],$$

where $|\alpha| < 1$.

   (a) Plot the pole-zero diagram and indicate the ROC for the system function.

   (b) Make a carefully labeled sketch of the magnitude $|H(e^{jw})|$. What are the maximum and the minimum values of its magnitude response? Use the pole-zero locations to explain why the frequency response looks as it does.

   (c) State whether the system is stable, causal, and minimum-phase system. Justify your answer.

   (d) Determine a closed-form expression for the frequency response $G(e^{jw})$ of the LTI discrete-time system characterized by an impulse response

$$g[n] = h[n] * h[n] * h[n],$$

   where $*$ denotes the convolution.

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10 points

4. Consider the cascade of the following three causal first-order LTI discrete-time systems:

$$H_1(z) = \frac{2+0.1z^{-1}}{1+0.4z^{-1}}, \quad H_2(z) = \frac{3+0.2z^{-1}}{1-0.3z^{-1}}, \quad H_3(z) = \frac{1}{1-0.2z^{-1}}.$$

   (a) Determine the transfer function of the overall system as a ratio of two polynomials in z^{-1}.

   (b) Determine the difference equation characterizing the overall system.

   (c) Develop the realization of the overall system with each section realized in direct form II.

   (d) Develop a parallel form I realization of the overall system.

   (e) Determine the impulse response of the overall system in closed form.

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10 points

5. A digital low-pass filter is required to meet the following specifications:

   - Passband gain between 0 dB and $-$ 0.5 dB
   - Passband edge $\pi/3$
   - Stopband attenuation of at least $-$ 40 dB
   - Stopband edge $\pi/2$

   The filter is to be designed by performing a bilinear transformation on an analog system function.

   (a) Determine what order Butterworth design must be used to meet the specifications in the digital implementations.
   Hint: You do not need to evaluate the system function or calculate the Butterworth cutoff frequency.

   (b) Which specifications (passband vs. stopband) will you meet exactly and which specifications (passband vs stopband) will you exceed. Justify your answer.

   (c) Repeat (b) if the Butterworth order in (a) was obtained by the impulse invariance method.

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5 points

6. For the lowpass filter specifications in Fig. 2, assume that $w_p = 0.63\pi$, $w_s = 0.65\pi$, $\delta_{p1} = \delta_{p2} = 0.05$, and $\delta_s = 0.1$. Design an FIR filter to meet these specifications by applying a Kaiser window to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff $w_c = 0.64\pi$. Find the values of β and M required to satisfy these specifications.

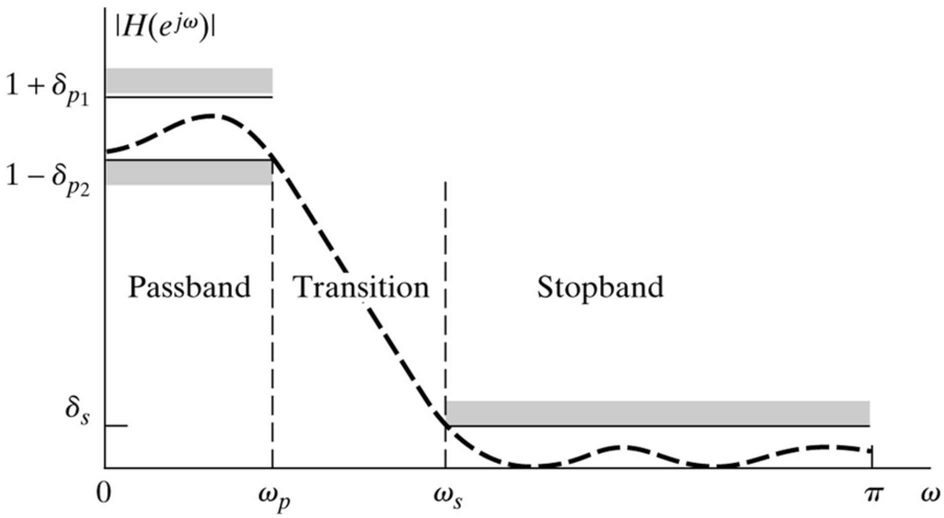

Figure 2: Lowpass filter tolerance scheme

5 points

7. Let $H(z)$ be the transfer function of a lowpass digital filter with a passband edge at w_p, stopband edge at w_s, passband ripple of δ_p, and stopband ripple of δ_s as indicated in Fig. 2. Consider a cascade of two identical filters each with a transfer function $H(z)$. What are the passband and stopband ripples of the cascade at w_p and w_s, respectively? Generalize the results for a cascade of M identical sections.

Good luck ...