University of Saskatchewan
Department of Electrical and Computer Engineering
EE 241 - Introduction to Electric Power Systems
Midterm Examination
February 13, 2019

Instructor: S.O. Faried **Duration: 2 hours**

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1. The following four loads are connected in parallel and supplied from a single-phase 600 V, 60-Hz source:
   Load A: 20 kVA, p.f. 0.5 lagging
   Load B: 10 kW, p.f. 0.9 leading
   Load C: 20 kVA, 12 kVAR lagging
   Load D: 10 kW, 10 kVA

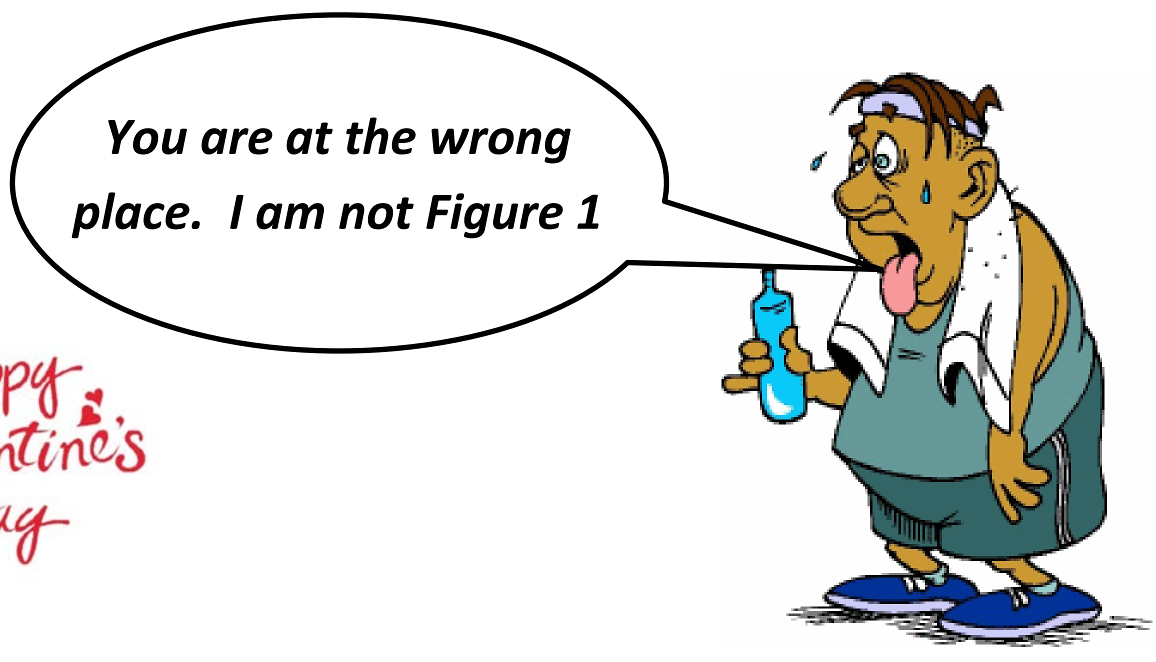
   (a) Draw the power triangle of the total load indicating the values of P, Q, S and the phase angle ϕ.

   (b) Determine the kVAR of a capacitor installed to improve the power factor to 0.95 lagging.

   (c) What is the value of the capacitor capacitance in μF.

(7 Marks)

2. A three-phase, **three-wire**, 208 V, ABC system supplies a Y-connected load in which $Z_A = Z_B = Z_C = 10\angle 30^{\circ}\Omega$ and a single-phase load $Z_{AB} = 30\angle 30^{\circ}\Omega$ as shown in Figure 1. Find the wattmeter reading.



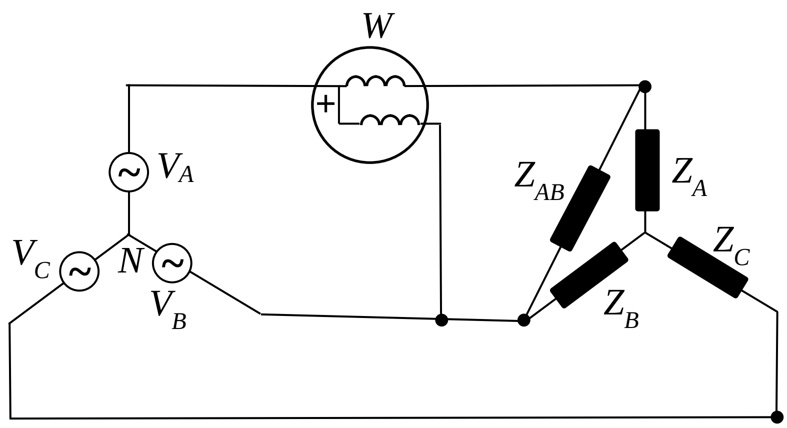


Figure 1.

(6 Marks)

3. A three-phase, **three-wire**, 380 V, ABC system supplies a Y-connected load of $Z_A = 10\angle 0^{\circ}\Omega$, $Z_B = 15\angle 60^{\circ}\Omega$, $Z_C = 10\angle -30^{\circ}\Omega$ through a three-phase cable with impedance $Z_{Cable} = 1\angle 75^{\circ}\Omega$ as shown in Figure 2. Find:

(a) The load phase voltages V_{LA}, V_{LB}, V_{LC}. Express the values in polar form ($Magnitude\angle angle^{\circ}$).

(b) The sum of the line-to-line voltages at the load ($V_{LAB}, V_{LBC}, V_{LCA}$).

(c) The total active power loss in the cable.

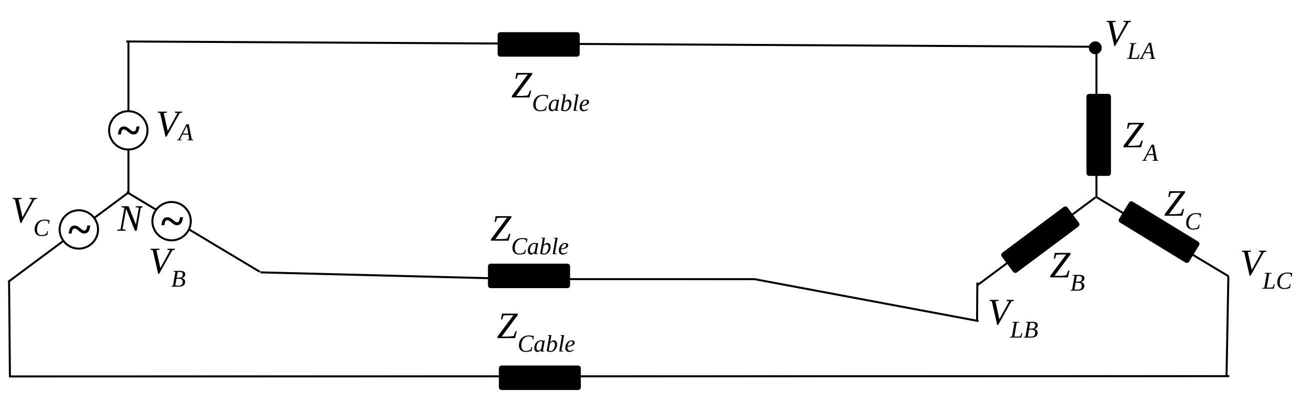


Figure 2.

(7 Marks)

Question # 1

$P = |S| \cos\phi$, $\frac{Q}{P} = \tan\phi$

$Q = |S| \sin\phi$, $P.F. = \cos\phi$

Load	P , KW	Q , KVAR
A	10	17.3205
B	10	-4.8432
C	16	12
D	10	0

Load B : $\phi = \cos^{-1} 0.9 = 25.8419^\circ \Rightarrow |S| = \frac{P}{\cos\phi} = 11.1111$ VA

$Q = |S| \sin\phi =$ KVAR

Load C : $12 = 20 \sin\phi \Rightarrow \phi = 36.8699^\circ$

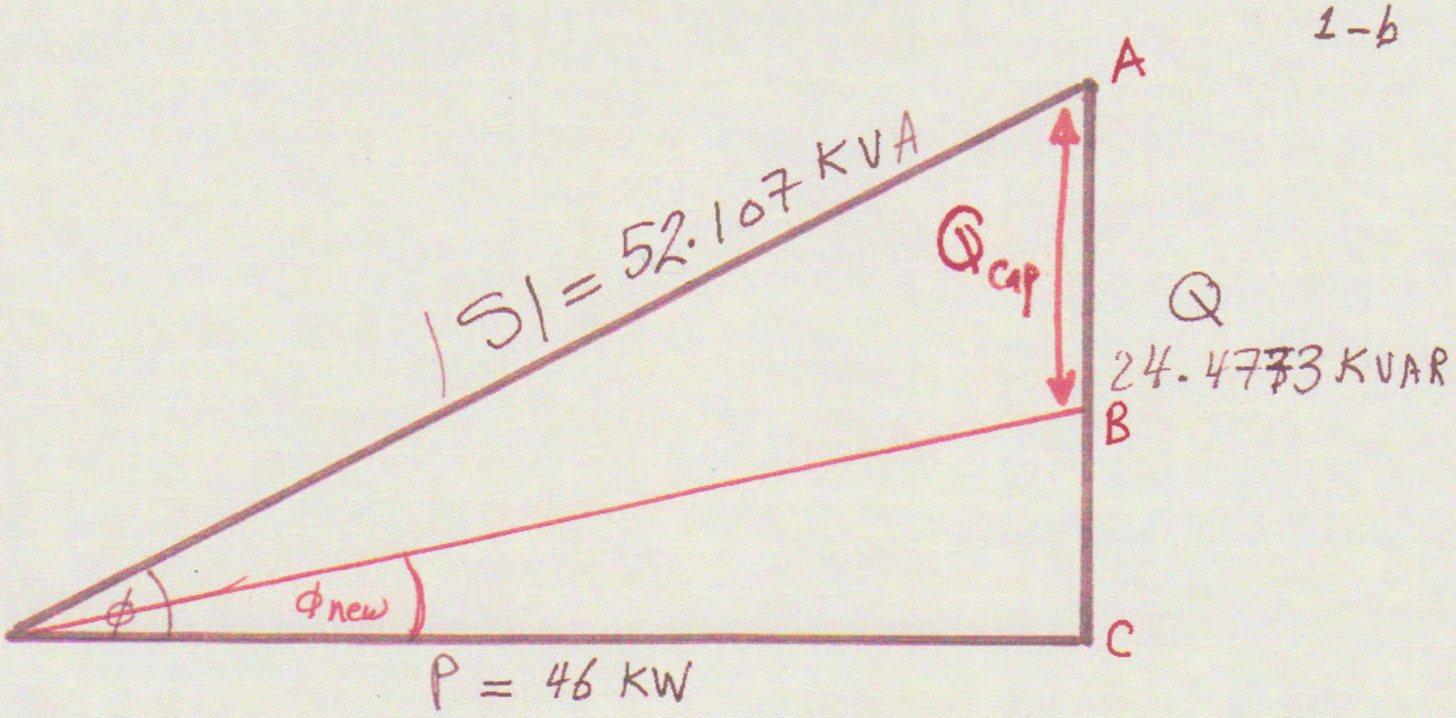
$P = 20 \cos\phi =$

$P_T = 46$ KW

$Q_T = 24.4773$ KVAR

$\phi = 28.0181^\circ$

$|S_T| = \sqrt{(46)^2 + (24.477)^2} = 52.1027$ KVA

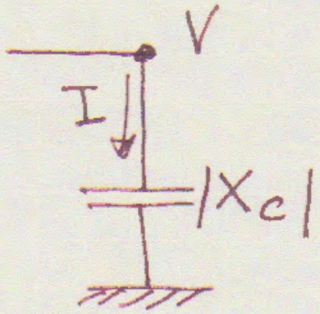


$\phi_{new} = \cos^{-1} 0.95 = 18.1949°$

$Q_{capacitor} = 24.4773 - 46 \tan\phi_{new} = \overline{AB}$

$Q_{capacitive} = 9.3578 \text{ KVAR}$

$Q_{cap} = |I|^2 |X_c|$ and $|I| = \frac{|V|}{|X_c|}$



$Q_{cap} = \frac{|V|^2}{|X_c|}$

$9357.8312 = \frac{(600)^2}{|X_c|}$

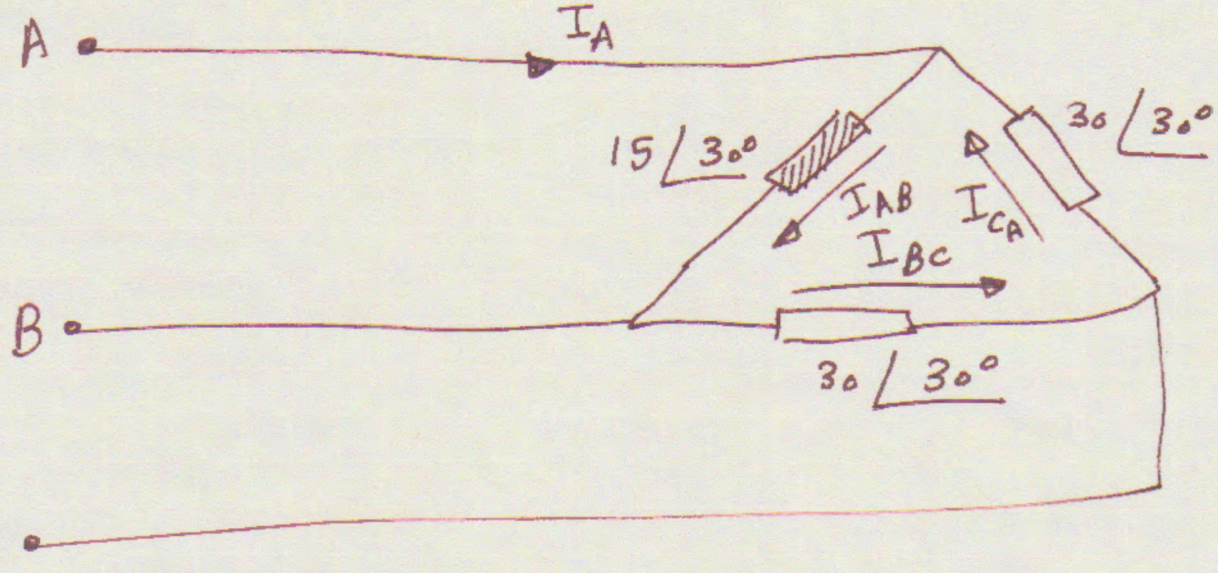
$|X_c| = 38.4705 \quad \Omega = \frac{1}{2\pi * 60 C}$

$C = 68.9512 \ \mu F$

Question # 2

Convert the Y to Δ ⇒ $Z_\Delta = 10\angle 30^\circ + 10\angle 30^\circ + \frac{10\angle 30^\circ * 10\angle 30^\circ}{10\angle 30^\circ}$

$Z_\Delta = 30\angle 30^\circ\ \Omega$



$V_{AB} = 208\angle 0^\circ$ V

$V_{BC} = 208\angle -120^\circ$ V

$V_{CA} = 208\angle 120^\circ$ V

$I_{AB} = \frac{V_{AB}}{15\angle 30^\circ} = \frac{208\angle 0^\circ}{15\angle 30^\circ} = 13.8667\angle -30^\circ$ A

$I_{BC} = \frac{208\angle -120^\circ}{30\angle 30^\circ} = 6.9333\angle -150^\circ$ A

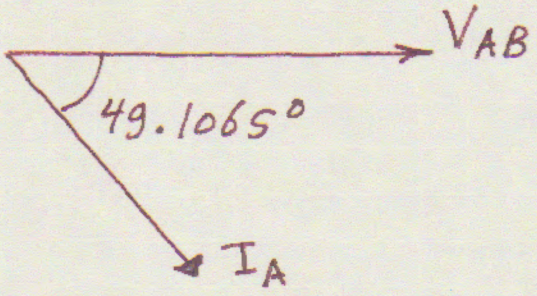
$I_{CA} = \frac{208\angle 120^\circ}{30\angle 30^\circ} = 6.9333\angle 90^\circ$ A

$$I_A + I_{CA} = I_{AB} \Rightarrow I_A = I_{AB} - I_{CA}$$

$$I_A = 13.8667 \angle -30^\circ - 6.9333 \angle 90^\circ$$

$$I_A = 18.3439 \angle -49.1065^\circ \text{ A}$$

$$V_{AB} = 208 \angle 0^\circ \quad \text{Reference}$$

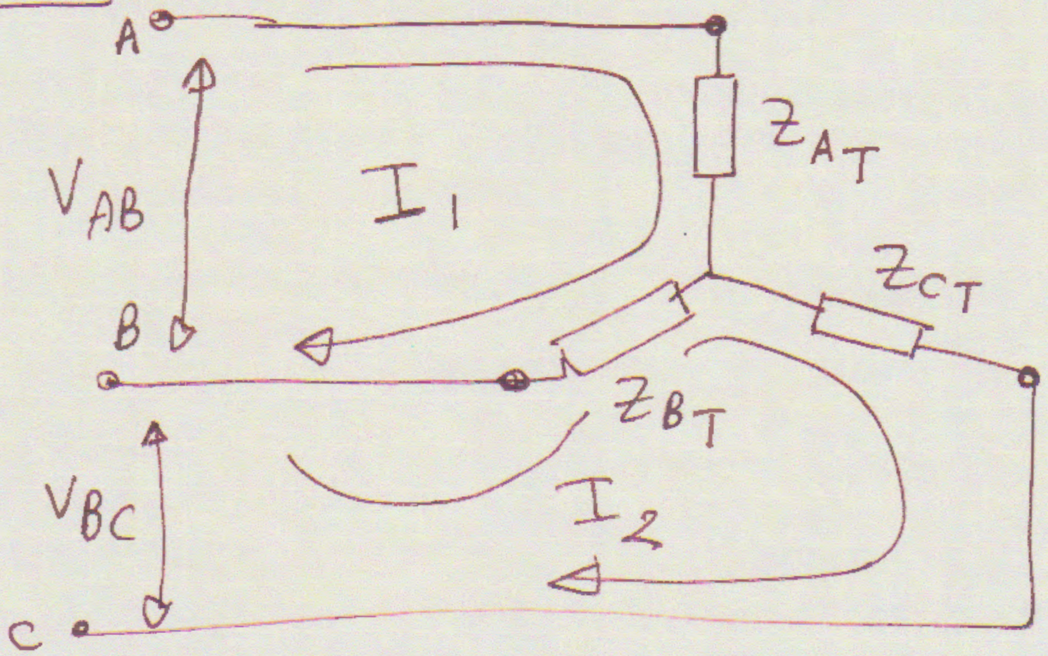


$$W_{Reading} = |V_{AB}|\,|I_A| \cos \widehat{V_{AB}\, I_A}$$

$$= 208 * 18.3439 \cos 49.1065^\circ$$

$$W_{Reading} = 2.4979 \text{ KW}$$

Question # 3



$$Z_{A_T} = 10\angle 0^\circ + 1\angle 75^\circ = 10.3042\angle 5.3789^\circ\ \Omega$$

$$Z_{B_T} = 15\angle 60^\circ + 1\angle 75^\circ = 15.968\angle 60.9287^\circ\ \Omega$$

$$Z_{C_T} = 10\angle -30^\circ + 1\angle 75^\circ = 9.789\angle -24.3371^\circ\ \Omega$$

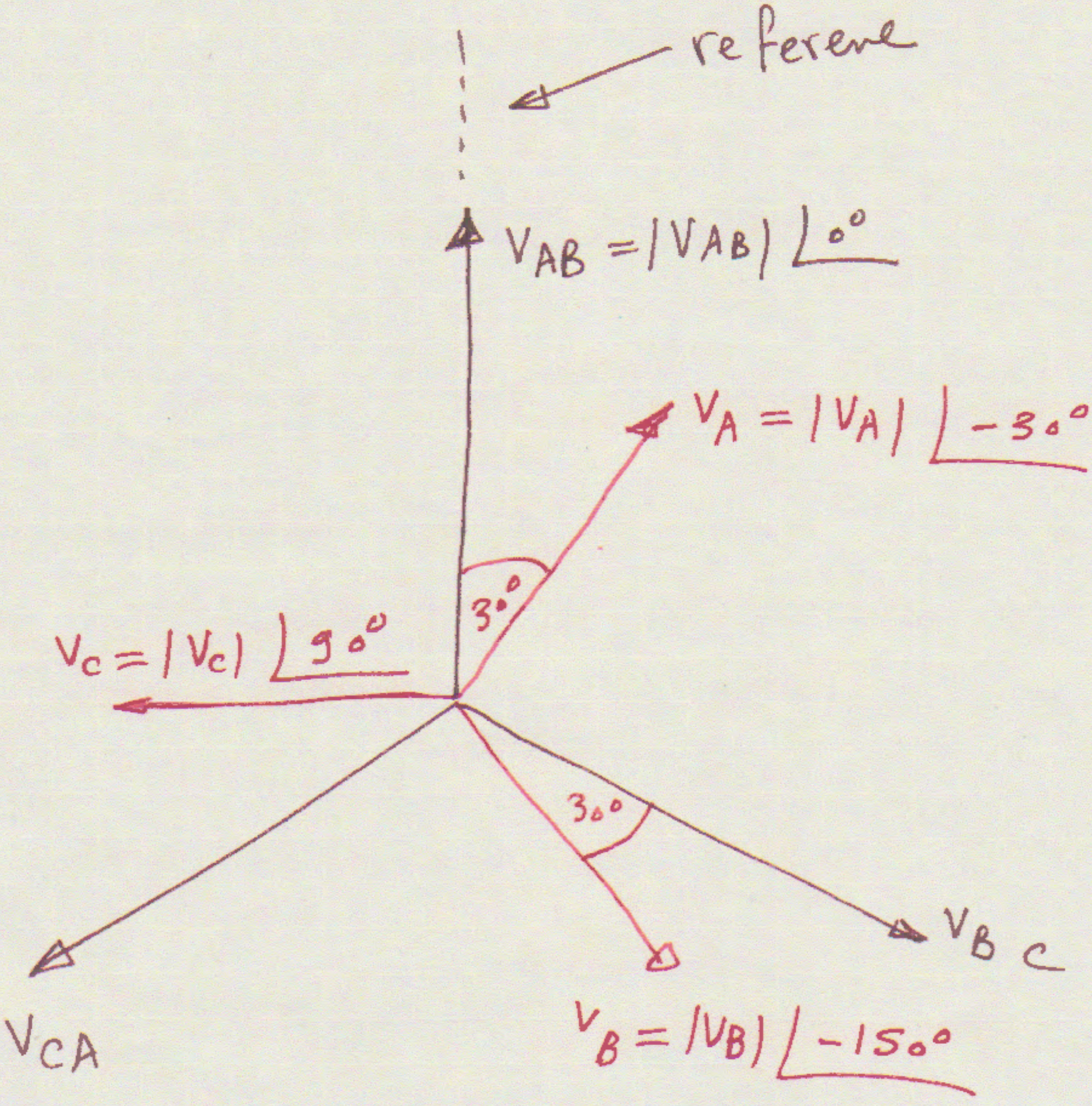
$$V_{AB} = 380\angle 0^\circ\ V$$

$$V_{BC} = 380\angle -120^\circ\ V$$

$$V_{AB} - Z_{A_T} I_1 - Z_{B_T} I_1 + I_2 Z_{B_T} = 0$$

$$V_{AB} = I_1 (Z_{A_T} + Z_{B_T}) - I_2 Z_{B_T}$$

$$V_{BC} - I_2 Z_{B_T} - I_2 Z_{C_T} + I_1 Z_{B_T} = 0$$


reference
$V_{AB} = |V_{AB}| \angle 0^\circ$
$V_A = |V_A| \angle -30^\circ$
30°
$V_c = |V_c| \angle 90^\circ$
30°
V_{BC}
V_{CA}
$V_B = |V_B| \angle -150^\circ$

$$V_{BC} = - I_1 Z_{B_T} + I_2 (Z_{B_T} + Z_{C_T})$$

$$\begin{bmatrix} 23.3946 \angle 39.6316^\circ & -15.968 \angle 60.9287 \\ -15.968 \angle 60.9287^\circ & 19.4063 \angle 30.7497 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 380 \angle 0^\circ \\ 380 \angle -120^\circ \end{bmatrix}$$

$$\Delta = 356.2741 \angle 36.3318^\circ \ \Omega$$

$$I_1 = \frac{\Delta 1}{\Delta} , \quad I_2 = \frac{\Delta 2}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 380 \angle 0^\circ & -15.968 \angle 60.9287 \\ 380 \angle -120^\circ & 19.4063 \angle 30.7497 \end{vmatrix}$$

$$\Delta_1 = 9{,}564.5156 \angle -8.6264^\circ$$

$I_1 = 26.8459 \angle -44.9582^\circ$ A

$$\Delta_2 = \begin{vmatrix} 23.3946 \angle 39.6316^\circ & 380 \angle 0^\circ \\ -15.968 \angle 60.9287^\circ & 380 \angle -120^\circ \end{vmatrix}$$

$\Delta_2 = 5626.3751 \angle -37.9652^\circ$

$I_2 = 15.7923 \angle -74.297^\circ$ A

$I_A = 26.8459 \angle -44.9582^\circ$ A

$I_B = 15.1967 \angle 165.6508^\circ$ A

$I_C = -15.7923 \angle -74.297^\circ$ A

$Z_{able} = 1 \angle 75^\circ = 0.2588 + j\, 0.9659\ \Omega$

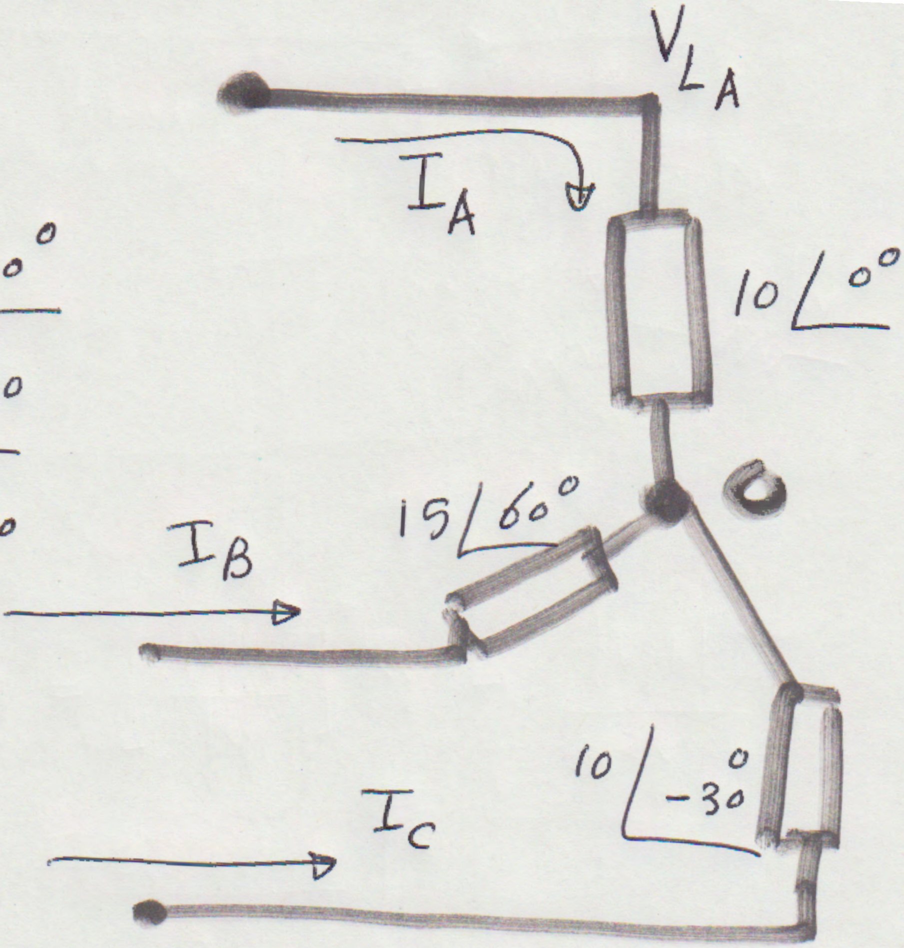
$P_{loss} = (26.8459)^2 * 0.2588 + (15.1967)^2 * 0.2588 + (15.7923)^2 * 0.2588$

$P_{loss} = 310.8288$ W

$V_{LA} = I_A * 10\angle 0°$

$V_{LB} = I_B * 15\angle 60°$

$V_{LC} = I_c * 10\angle -30°$



$V_{LA} = 268.459\angle -44.9582°$ V

$V_{LB} = 227.9505\angle -134.3492°$ V

$V_{LC} = 151.967\angle 75.703°$ V

Line-to-line voltages at the load

$V_{LAB} = V_{LA} - V_{LB} = 350.3295\angle -4.3685°$ V

$V_{LBC} = V_{LB} - V_{LC} = 367.4557\angle -122.3962°$ V

$V_{LCA} = V_{LC} - V_{LA} = 369.8295 \angle 114.3424^\circ$ V

$V_{LAB} + V_{LBC} + V_{LCA} = 0$

The sum of the line-to-line voltages is always zero (balanced or unbalanced load)