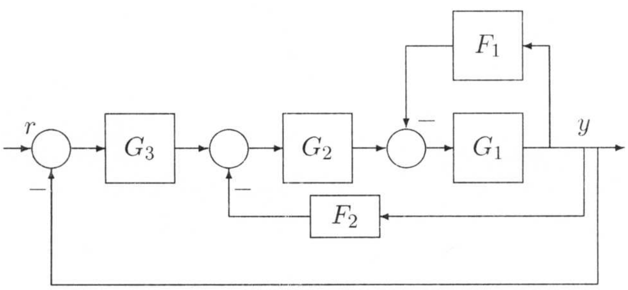
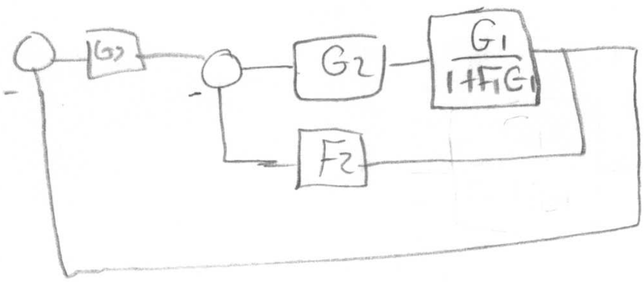
24.3

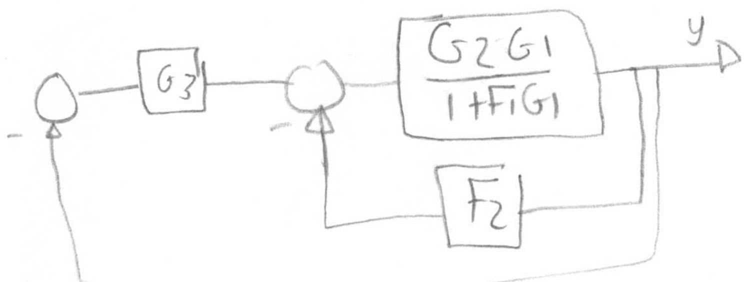
EE481.3 Mid-term test, Control systems

4 Questions, Duration 75 minutes.
You may use a formula sheet, and a calculator.
Attempt all the questions. Justify your answers.

(6 marks) 1) Find the transfer function of the following system.

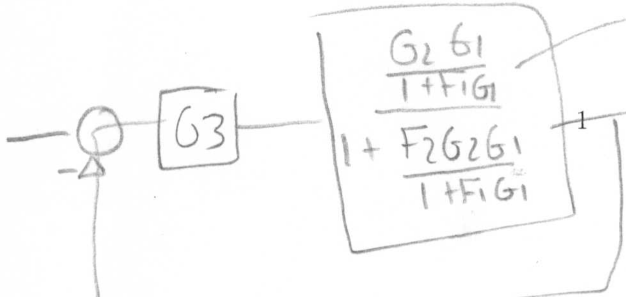




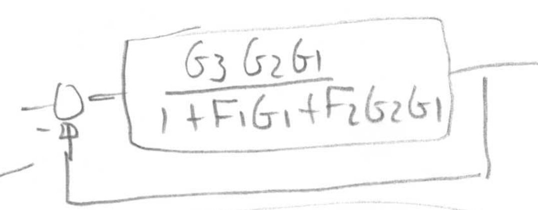


6.0

$$\frac{G_2G_1}{1+F_1G_1+F_2G_2G_1}$$



$$\frac{\frac{G_2G_1}{1+F_1G_1}}{1+\frac{F_2G_2G_1}{1+F_1G_1}}$$

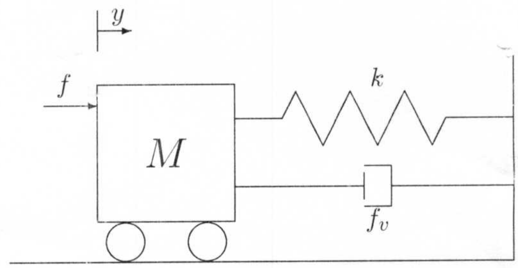


$$\frac{G_3G_2G_1}{1+F_1G_1+F_2G_2G_1}$$

$$\frac{\frac{G_3G_2G_1}{1+F_1G_1+F_2G_2G_1}}{1+G_3G_2G_1/1+F_1G_1+F_2G_2G_1} = \boxed{\frac{y}{r}=\frac{G_3G_2G_1}{1+F_1G_1+F_2G_2G_1+G_3G_2G_1}}$$

(6 marks) 2- What is the equation describing the system shown below. Find the transfer function Y/F?

Take M =2.0kg, $k = 8N/m$ and $f_v = 4Ns/m$. What is the steady state value of the output y, when the input force is 1N. What is the settling time to within 2 percent of the final value. Is the response of the system overdamped or Underdamped? ? If we can replace the shock-absorber(damper), how should we choose a new shock-absorber, so that the resulting mechanical system has a step response with a percent-overshoot of 5%.



INPUT

input force = 1N on (t) $= \frac{1}{s}$

$$\Sigma F = 0 = f - y(s)(k + f_v s + Ms^2)$$

$$\boxed{\frac{y(s)}{f} = \frac{1}{k + f_v s + Ms^2}}$$

M = 2 kg
k = 8 N/m
f_v = 4 Ns/m

final value (input)

$$y(\infty) = \lim_{s \to 0} s y(s)$$

$$y(s) = \frac{1/s}{K + f_v s + Ms^2}$$

$$\lim_{s \to 0} s y(s) = \frac{1}{k + f_v s + Ms^2} = \frac{1}{k} = \boxed{1/8 \text{ m}}$$

$0 < \zeta < 1$ underdamped

$$y(s) = \frac{1}{8 + 4s + 2s^2} = \frac{0.5}{s^2 + 2s + 4} = \frac{\frac{1}{8}(4)}{s^2 + 2s + 4}$$

$\omega_n^2 = 4$ $\quad \zeta = \frac{2}{2\omega_n} = \frac{2}{2 \cdot 2} = 0.5$

$\omega_n = 2$

$$T_s = \frac{4}{\zeta \omega_n} = \frac{4}{2 \cdot 0.5} = \boxed{4 \text{ sec}}$$

$$\frac{\frac{1}{8}(4)}{(s + 1 + j1.73)(s + 1 - j1.73)}$$

two complex poles $\boxed{\text{underdamped}}$

%OS = 5

$$\zeta = \frac{-\ln(5/100)}{\sqrt{\pi^2 + \ln^2(5/100)}} = 0.69$$

5.7

$f_v = 2\zeta\omega_n M$

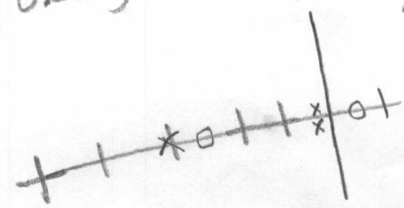
$f_v = 2(0.69)(2) = \boxed{2.76}$

(6 marks) 3. Take the system $G(s) = \frac{(2s+5)(-s+0.5)}{(s+3)(s^2+0.1s+0.01)}$. Discuss the stability of the system. Find a low-order approximation for this system. Sketch the step response of the approximated system and discuss the accuracy of approximating the response of the system using the simplified low-order system.

$$\frac{(2s+5)(-s+0.5)}{(s+3)(s^2+0.1s+0.01)} = \frac{(2s+5)(-s+0.5)}{(s+3)(s+0.05+j0.087)(s+0.05-j0.087)}$$

$$= -\frac{(2s+5)(s-0.5)}{(s+3)(s+0.05+j0.087)(s+0.05-j0.087)}$$

Splane



$\boxed{\text{all poles are on left hand side of Splane so Stable}}$

$3 \gg 0.05 \cdot 10 = 0.5$ So can ignore pole @ -3 but need to Consider DC gains from terms

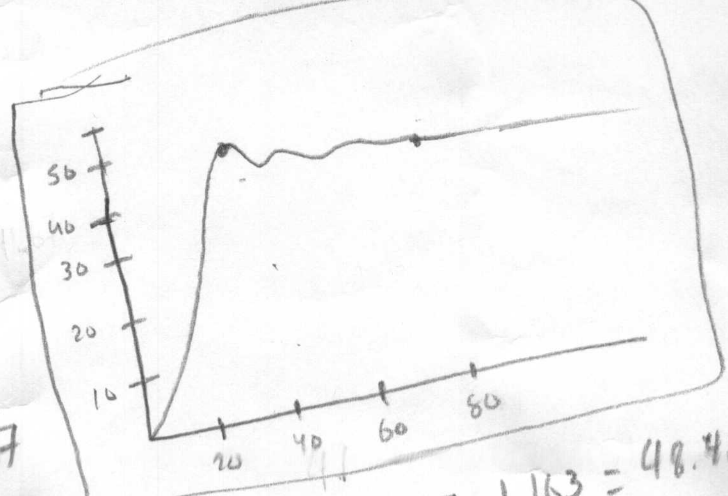
2.5 & zero @ -2.5

0.5 & zero @ 0.5

5.6

$$\frac{5/2 \cdot 1/2}{3} \quad \frac{1}{(s^2+0.1s+0.01)}$$

$$\frac{Y}{R} = \boxed{\frac{5}{12(s^2+0.1s+0.01)}}$$



$$y(\infty) = \lim_{s\to 0} sY(s) = \frac{5 \cdot \cancel{s} \cdot \cancel{\frac{1}{s}}}{12(0+0+0.01)} = \cancel{41}.67$$

$41.67 \cdot 1.163 = 48.46$

$$\omega_n^2 = 0.01 \qquad PO = e^{-\left(\frac{0.5\pi}{\sqrt{1-0.5^2}}\right)} 100 = 16.3\%$$

$\omega_n = 0.1$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{0.1\sqrt{1-0.5^2}} = 36.28$$

$0.1 = 2\zeta\omega_n$

$$\zeta = \frac{0.1}{0.1 \cdot 2} = \frac{1}{2} = 0.5 \qquad T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.1 \cdot 0.5} = 80$$

With the Added poles the system would react abit quicker but since they were 10 bigger they would hit their S.S way be fore the two approx S.S poles

(7 marks) 4. Take the system $G(s) = \frac{1}{s^3+0.3s^2+0.03s+0.002}$ Find a state space representation for this system. What are the eigen-values of the system matrix A. Design a state-feedback such that the resulting system has a settling time of 10 seconds and a p.o. of 5%.

$$G(s) = \frac{y}{r} = \frac{1}{s^3+0.3s^2+0.03s+0.002}$$

$$Y(s) = X_1(S)$$

$$X_2(S) = \dot{X}_1(S)$$

...

$$y(s^3+0.3s^2+0.03s+0.002) = r$$

$$\boxed{\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -0.002 & -0.03 & -0.3\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} r \qquad y = [1\ 0\ 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + 0[r]}$$

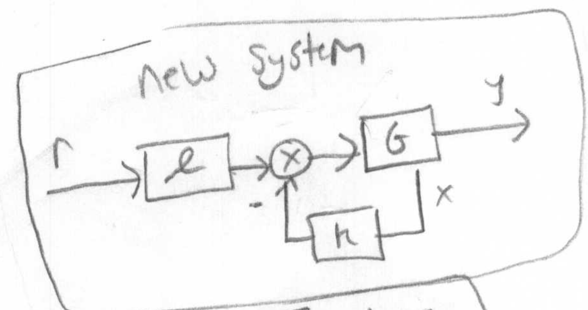
eigen values = $\det(\lambda I - a)$

$$\det\left(\begin{bmatrix}\lambda & -1 & 0\\ 0 & \lambda & -1\\ 0.002 & 0.03 & \lambda+0.3\end{bmatrix}\right) = \lambda^3 + 3\lambda^2 + 0.3\lambda + 0.002$$

$$\lambda = (-0.05 - j0.0866)$$

$$\lambda = (-0.05 + j\,0.0866)$$

$$\lambda = -0.2$$



design state feed back

$Ts = 10 \quad p.o. = 5\%$

$$\beta = \frac{-\ln(5/100)}{\sqrt{\pi^2 + \ln^2(5/100)}} \approx 0.69$$

new State space

$$\begin{array}{c} \\ \dot{X}_1\\ \dot{X}_2\\ X_3\end{array}\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1.682 & -4.3384 & -5.8004\end{pmatrix}\begin{array}{c}X_1\\ X_2\\ X_3\end{array}$$

$$-a_0 - k_0 \qquad -a_1 - k_1 \qquad -a_2 - k_2$$

$$\boxed{\begin{aligned} k_0 &= 1.68\\ k_1 &= 4.35\\ k_2 &= 5.5004\end{aligned}}$$

$$\boxed{l = \frac{0.3364}{1}}$$

$$Ts = \frac{4}{\beta \omega n} \qquad \omega n = \frac{4}{Ts\beta} = \frac{4}{10 \cdot 0.69} = 0.58$$

4

$$\frac{0.3364}{(s+0.4+j0.42)(s+0.4-j0.42)(s+5)}$$

$$\frac{\omega n^2}{s^2 + 2\beta\omega n s + \omega n^2} = \frac{0.3364}{s^2 + 0.8004 s + 0.3364} = \frac{0.3364}{s^3 + 5.8004 s^2 + 4.33845 + 1.682}$$

$$= \frac{0.3364}{(s+0.4+j0.42)(s+0.4-j0.42)}$$

Add pole @ks1
$10 \cdot 0.4 = 4$ So pick 5

7.0