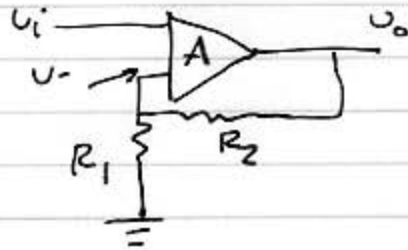


Controls Review

Sensitivity:

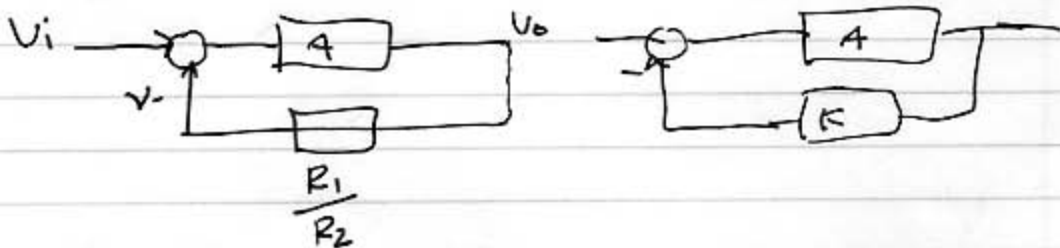
$$S_p^T = \frac{\frac{\partial T}{\partial P}}{\frac{T}{P}}$$



$$A_1 = 10000$$

$$A_2 = 5000$$

$$\frac{R_1}{R_2} = 0.1$$



$$T_1 = \frac{A}{1 + AK} = \frac{10^4}{1 + 10^4 \cdot 0.1} = \frac{1000}{1 + 10^3} \approx \frac{10000}{10001} \approx 10$$

$$T_2 = \frac{5000}{5001} \approx 10$$

$$\frac{\partial T}{\partial P} = \frac{1 + AK - kA}{(1 + kA)^2} = \frac{1}{(1 + kA)^2}$$

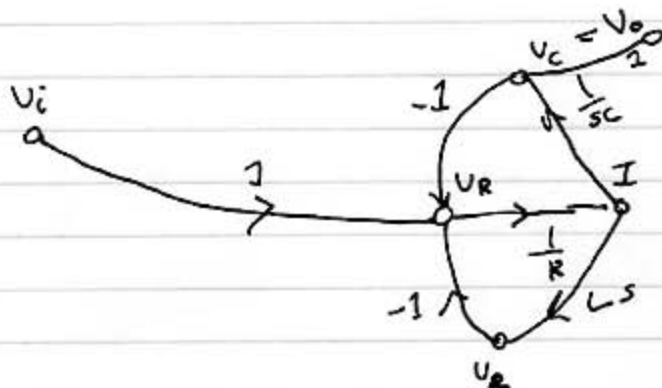
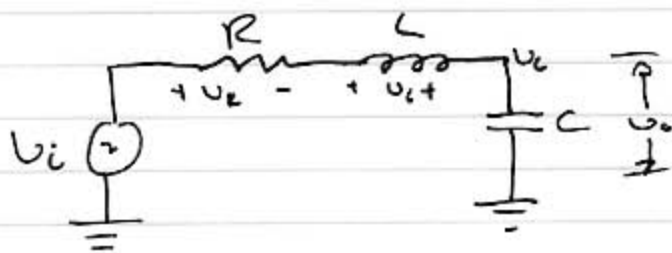
$$\frac{T}{A} = \frac{1}{1 + AK}$$

$$S_A^T = \frac{1}{1 + kA} = \frac{1}{1 + 0.1 \cdot 10^4} = \frac{1}{1001} \approx 0.001 \quad \frac{1}{501}$$

$$s_A^T = \frac{\frac{\Delta T}{T}}{\frac{\Delta A}{A}} = \Delta T = S_A^T \Delta A \frac{T}{A}$$

$$= 0.001 (-5000) \frac{1}{1 + 10^4 (0.1)} \approx 0.005$$

If k changes then the TFcm is changed more \therefore more sensitive.



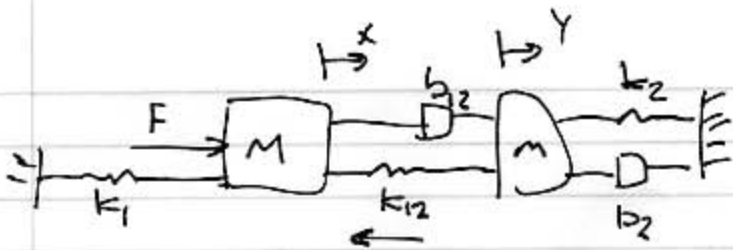
$$P_1 = \frac{1}{RCs}$$

$$D_1 = 1$$

$$L_1 = 1 - \frac{Ls}{R}$$

$$L_2 = -\frac{1}{sCR}$$

$$\frac{V_c}{V_i} = \frac{\frac{1}{RCs}}{1 - \left[\frac{-Ls}{R} - \frac{1}{sCR} \right]} = \frac{1}{RCs + RLCs^2 + 1}$$



$$M: F - k_{12}(x-y) - b_{12}(\dot{x}-\dot{y}) - k_1 x = M \ddot{x}$$

$$m: k_{12}(y-x) - b_{12}(\dot{y}-\dot{x}) - b_2 \dot{y} - k_2 y = m \ddot{y}$$

~~$$F = M \ddot{x} + k_{12}(x-y) + b_{12}(\dot{x}-\dot{y}) + k_1 x$$~~

$$F = M \ddot{x} + b_{12} \dot{x} + (k_{12} + k_1)x - (b_{12} \dot{y} + k_{12} y)$$

$$0 = -(b_{12} \dot{y} + k_{12} x) m \dot{y} + (b_2 + b_{12}) \dot{y} + (k_2 + k_{12}) y$$

$$\begin{pmatrix} \hat{F}(s) \\ 0 \end{pmatrix} = \begin{pmatrix} M s^2 + \overset{A_{11}}{b_{12} s + k_{12} + k_1} & \overset{A_{12}}{-(b_{12} s + k_{12})} \\ \overset{A_{21}}{-(b_{12} s + k_{12})} & \overset{A_{22}}{m s^2 + (b_2 + b_{12}) s + k_2 + k_1} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

at steady state no change of time $\dot{x}, \dot{y} = 0$ x stays constant

so for steady state remove the dampers

$$F - k_{12}(x-y) - k_1 x = 0 \quad (1) \quad \text{and solve}$$

$$-k_{12}(y-x) + k_2 y = 0 \quad (2)$$

or
find $T(s)$ from F to x and set $s=0$

$$\frac{X}{F(s)} = \dots$$

~~$$\begin{pmatrix} F(s) \\ 0 \end{pmatrix}$$~~

$$\begin{pmatrix} X(s) \\ Y(s) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}^{-1} \begin{pmatrix} F(s) \\ 0 \end{pmatrix}$$

Hibroy

$$\begin{pmatrix} X(s) \\ Y(s) \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} A_{22} & -A_{12} \\ -A_{12} & A_{11} \end{pmatrix} \begin{pmatrix} F(s) \\ 0 \end{pmatrix}$$

$$= \frac{\begin{pmatrix} A_{22} \hat{F}(s) \\ -A_{12} \hat{F}(s) \end{pmatrix}}{\Delta} \quad X(s) =$$

$$\frac{X(s)}{\hat{F}(s)} = \frac{A_{22}(s)}{A_{11}A_{22} - A_{12}^2}$$

$$\frac{Y(s)}{\hat{F}(s)} = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2}$$

if $F = 1N$ find X at steady state

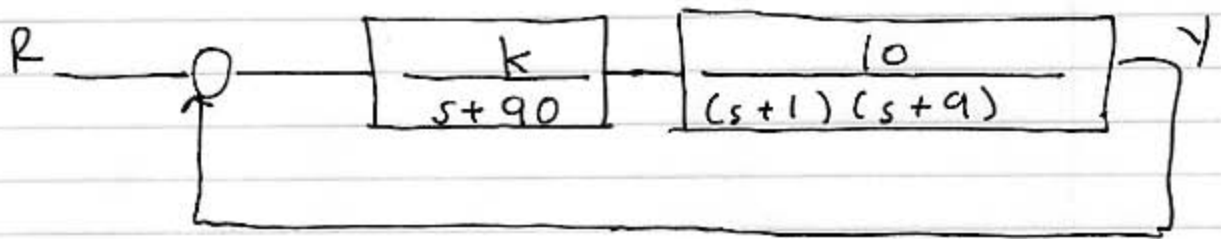
$$X(s) \Big|_{s=0} = \frac{A_{22}(s) \Big|_{s=0}}{(A_{11}A_{22} - A_{12}^2) \Big|_{s=0}} \quad \Leftarrow \text{steady state value}$$

$$= \frac{k_2 + k_{12}}{(k_2 + k_1)(k_2 + k_1) - k_{12}^2}$$

$$\text{if } k_1 = k_2 = k_{12} = 2$$

$$= \frac{2}{4-1} = \frac{2}{3}$$

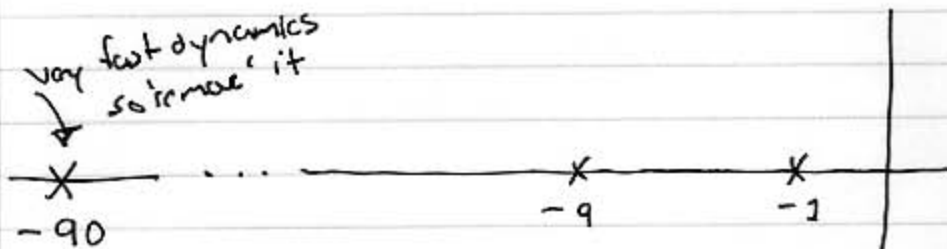
$$= \frac{k_2 + k_{12}}{k_2 k_1 + k_2 k_2 + k_1 k_{12}}$$



Find k such that $e_{ss} < 0.12$ for unit step input
and $p.o \leq 15\%$.

$$\frac{Y}{R} = \frac{10k}{(s+90)(s+1)(s+9)}$$

but we want the DC
gain to be the same
while removing an s



Find steady state error: uses the plant

$$k_p = \lim_{s \rightarrow 0} P(s) = \frac{k}{(s+90)(s+1)(s+9)} = \frac{10k}{810} = \frac{k}{81}$$

$$e_{ss}(r) = \frac{1}{1+k_p} = \frac{1}{1 + \frac{k}{81}} < 0.12$$

$$0.88 < \frac{k}{81} < 0.12$$

~~$$k > 594$$~~

~~PO = 15%~~
now for p.o. $\leq 15\%$ \Rightarrow use close loop TF

$$\zeta \approx 0.52$$

$$T = \frac{k}{9} \frac{1}{s^2 + 10s + 9} = \frac{\frac{k}{9}}{s^2 + 10s + (9 + \frac{k}{9})}$$

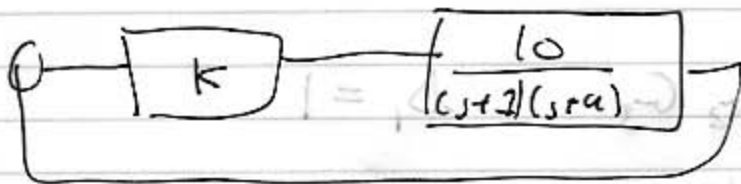
\hookrightarrow substitute this for ω_n^2

$$2\zeta\omega_n = 10$$
$$\omega_n = \frac{10}{2(0.52)} = \sqrt{\frac{9 + \frac{k}{9}}{9}}$$

$$\left(\left(\frac{5}{0.52} \right)^2 - 9 \right) 9 = k \leq 751$$

$$594 < k < 751$$

Then simulate the real ~~3rd~~ 3rd order system



↳ TYPE 0
input & output @
steady state

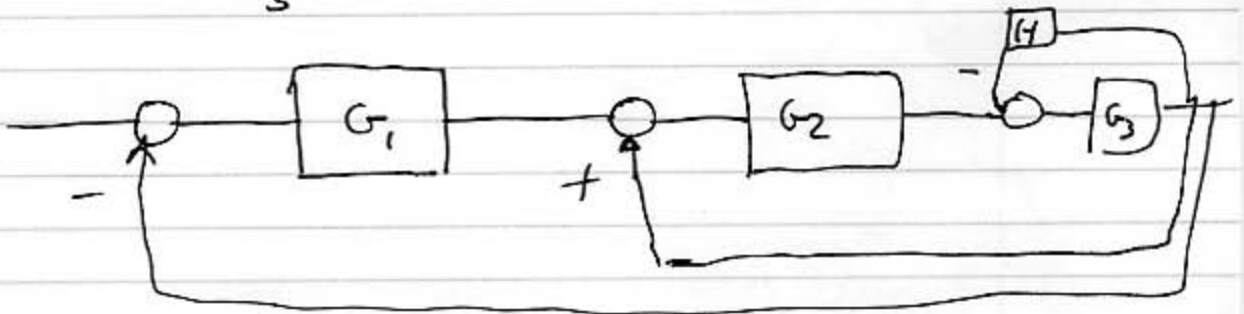
if you have $\frac{k}{s}$ it is a type I system

conditions: stable

if you have $\frac{k}{s} + k_2$ is a type I system still

$$\Rightarrow \frac{k + k_2 s}{s}$$

if you have $\frac{k_1 + k + k_2 s}{s}$ is still type I



$$\frac{G_1 \frac{G_2 H}{1+HG_3}}{1 - \frac{G_2 H}{1+HG_3}} = \frac{G_1 \frac{G_2 H}{1+HG_3}}{[1 - \frac{G_2 H}{1+HG_3}] + G_1 G_2 \frac{H}{1+HG_3}}$$

$$= \frac{G_1 G_2 H}{(1+HG_3) [1 - \frac{G_2 H}{1+HG_3}] + G_1 G_2 H} = \frac{G_1 G_2 H}{1+HG_3 - \frac{G_2 H}{1+HG_3}} = \text{omg!} \neq$$

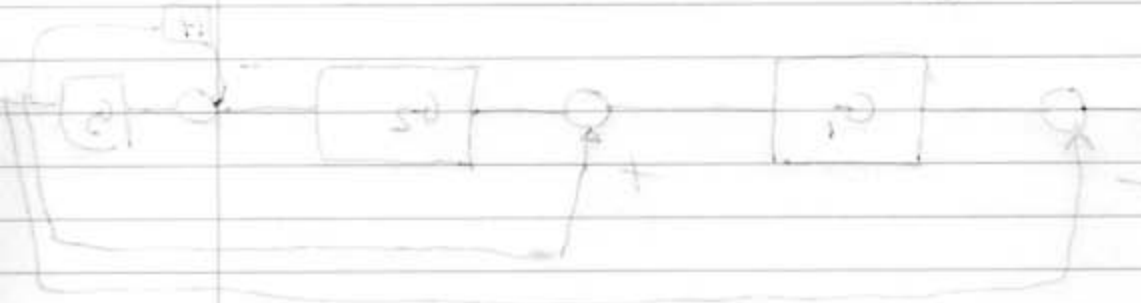
$$P1 = G_1 G_2 G_3 \Delta_1 = 1$$

$$\Delta = -G_3 H + G_2 G_3 - G_1 G_2 G_3$$

$$T(s) = \frac{G_1 G_2 G_3}{1 + G_3 H - G_2 G_3 + G_1 G_2 G_3}$$

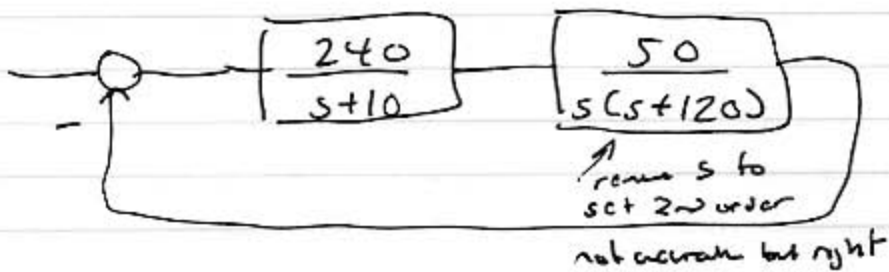
they are all factoring!

if you have a feedback loop in a system...



$$Y(s) = \frac{G_1(s) G_2(s) G_3(s) R(s)}{1 + G_3(s) H(s) - G_2(s) G_3(s) + G_1(s) G_2(s) G_3(s)}$$

Last Year

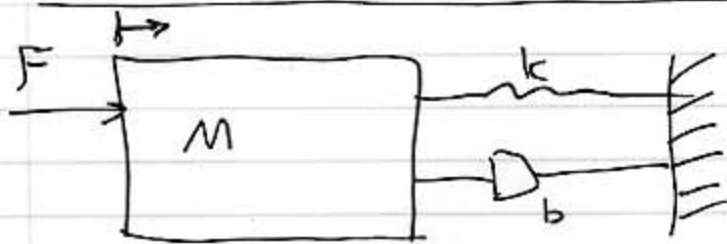


Find p.o., T_s , T_p , steady state errors to ramp + step inputs

steady state error to steps: Type I \Rightarrow 0 error

$$K_v = \frac{(240)(50)}{(10)(120)} = 10$$

$$T(s) = \dots$$



What is the steady state value of the output y if $M=2.4$ $k=30$ $b=10 \text{ N s/m}$ $F=1 \text{ N}$

$$F = ky$$

$$1 = 30y \quad y = \frac{1}{30}$$

$$(Ms^2 + bs + k)Y = F$$

$$\frac{Y}{F} = \frac{1}{Ms^2 + bs + k} \quad \frac{Y}{F} \Big|_{s=0} = \frac{1}{30}$$

$F=1$