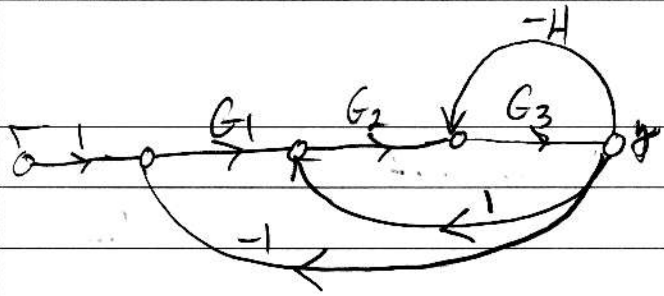
1)



$$\frac{Y}{R} = \frac{\Sigma P_k \Delta_k}{\Delta}$$

$P_1 = G_1 G_2 G_3$

$L_1 = -G_3 H$

$L_2 = G_2 G_3$

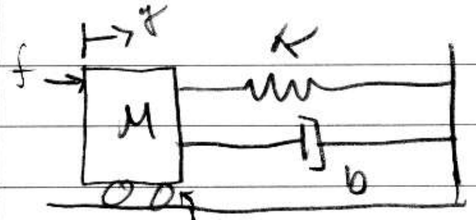
$L_3 = -G_1 G_2 G_3$

$\Delta_1 = 1$

$$\frac{Y}{R} = \frac{G_1 G_2 G_3}{1 - G_2 G_3 + G_3 H + G_1 G_2 G_3}$$

60

2)



assume no friction

Find Y/F

$M = 2.4\,kg$ $\quad k = 30\,N/m$ $\quad b = 10\,Ns/m$.

No initial conditions

$\frac{d^2y}{dt^2} = s^2 Y(s)$ ☆

$\frac{dy}{dt} = sY(s)$

FBD

$$M\frac{d^2y}{dt^2} = F - ky - b\frac{dy}{dt}$$

$$Ms^2 Y(s) = F(s) - kY(s) - bsY(s)$$

$$F(s) = Y(s)\left[Ms^2 + k + bs\right]$$

$$\boxed{\frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + bs + k}} = \frac{1}{2.4s^2 + 10s + 30} = \boxed{\frac{0.417}{s^2 + 4.17s + 12.5}}$$

Steadystate

$$M\cancel{\frac{d^2y}{dt^2}} = F - ky - \cancel{b\frac{dy}{dt}}$$

$$F = ky$$

$$y = \frac{F}{k} = \frac{1N}{30\frac{N}{m}}$$

$$\boxed{y = 33.3mm}$$

60

$T_s = \frac{4}{\zeta\omega_n}$ since $\delta = 2\%$

$2\zeta\omega_n = +4.17$ from Tx Fn

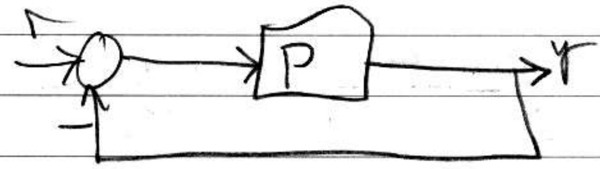
$\zeta\omega_n = 2.085$

$\rightarrow T_s = \frac{4}{2.085}$

$$\boxed{T_s = 1.918s}$$

Discuss stability & e_{ss} for step & ramp I/P

3)



a) $P(s) = \dfrac{161}{(s+5)(s+4)} = \dfrac{161}{s^2+9s+20}$

$\dfrac{Y}{r} = \dfrac{P}{1+P} = \dfrac{\frac{161}{(s+5)(s+4)}}{1+\frac{161}{(s+5)(s+4)}} = \dfrac{161\,(s+5)(s+4)}{(s+5)(s+4)\left[(s+5)(s+4)+161\right]}$

$\dfrac{Y}{r} = \dfrac{161}{s^2+9s+181}$ $\qquad s_{1,2} = \dfrac{-9 \pm \sqrt{(9)^2 - 4(1)(181)}}{2}$

$s_{1,2} = \dfrac{-9 \pm j\sqrt{643}}{2}$

System is Stable since $s_{1,2}$ both in LHS plane and no unstable pole zero cancellations. ✓

Step I/P e_{ss}

Type 0 system

$\therefore$ e_{ss} to step is $e_{ss} = \dfrac{1}{1+K_p}$

where $K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \dfrac{161}{s^2+9s+20} = \dfrac{161}{20} = 8.05$

$e_{ss} = \dfrac{1}{1+8.05} = 0.11 \Rightarrow \boxed{e_{ss_{\text{step I/P}}} = 11\%}$ ✓

Not to bad for stability.

Ramp I/P e_{ss}

Type 0 system

$\therefore$ e_{ss} to ramp I/P is ∞

$\boxed{e_{ss} = \infty}$ ✓ System will never reach a steady state with a ramp I/P so not stable in that case

$Y(s) = \dfrac{1}{s^2}\,\dfrac{161}{(s^2+9s+181)}$ using F.V.T. $\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,\dfrac{161}{s^2(s^2+9s+181)} = \boxed{\infty}$

3)

b) $P(s) = \frac{192}{s(s+5)(s+4)} = \frac{192}{s(s^2+9s+20)} = \frac{192}{s^3+9s^2+20s}$

$T(s) = \frac{Y(s)}{r(s)} = \frac{P(s)}{1+P(s)} = \frac{\frac{192}{s(s+5)(s+4)}}{1+\frac{192}{s(s+5)(s+4)}} = \frac{192}{s(s+5)(s+4)} \cdot \frac{s(s+5)(s+4)}{s(s+5)(s+4)+192}$

$\frac{Y(s)}{r(s)} = \frac{192}{s(s+5)(s+4)+192} = \frac{192}{s^3+9s^2+20s+192}$

Routh Hurwitz

s_3	1	20	0
s_2	9	192	0
s_1	-1.333	0	
s_0	192	0	

$\frac{1(192) - 9(20)}{-9} = -1.333$

System is not stable

Since all left hand column entries don't have the same sign the system has poles in RHS and is unstable.

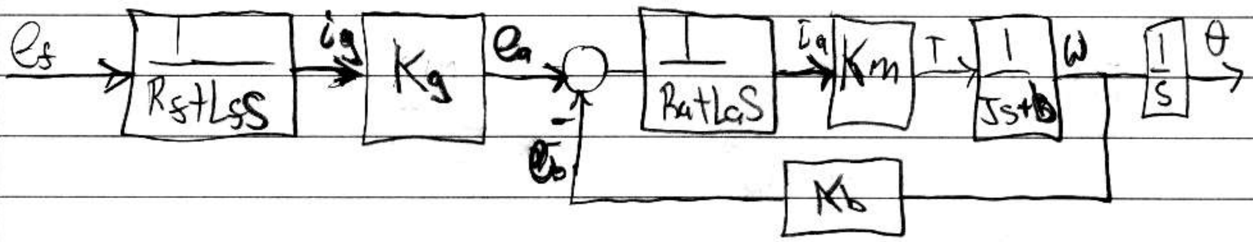
Since unstable you don't care about ess since it is unstable

6.0

4) Armature Controlled DC motor.

generator constant speed ω_g. obtain $\frac{\theta(s)}{e_f(s)}$.

Assume $e_a = K_g i_g$



$$\frac{\theta(s)}{e_f(s)} = \frac{\left(\frac{1}{R_f+L_fs}\right) K_g \left(\frac{1}{R_a+L_as}\right) K_m \left(\frac{1}{Js+b}\right) \frac{1}{S}}{1+\left(\frac{1}{R_a+L_as}\right) K_m \left(\frac{1}{Js+b}\right) K_b}$$

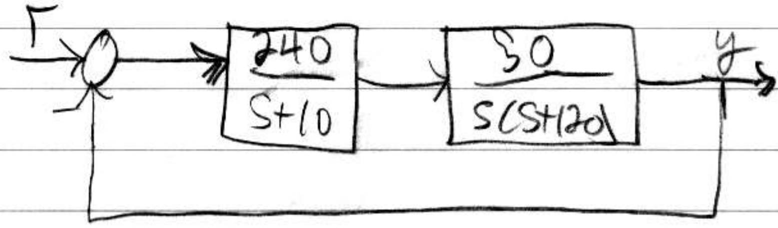
$$\frac{\theta(s)}{e_f(s)} = \frac{K_g K_m}{S(R_f+L_fs)\cancel{(R_a+L_as)}\cancel{(Js+b)}} \; \frac{\cancel{(R_a+L_as)}\cancel{(Js+b)}}{(R_a+L_as)(Js+b)+K_mK_b}$$

$$\boxed{\frac{\theta(s)}{e_f(s)} = \frac{K_g K_m}{S(R_f+L_fs)\left[(R_a+L_as)(Js+b)+K_mK_b\right]}}$$

6.0

$(-4.55246153506, -8.86198166148)($

5)



Find second order approx.

$$\frac{y}{r} = \frac{\dfrac{12000}{S(S+10)(S+120)}}{1+\dfrac{12000}{S(S+10)(S+120}} = \frac{12000}{\cancel{S(S+10)(S+120)}} \cdot \frac{\cancel{S(S+10)(S+120)}}{S(S+10)(S+120)+12000}$$

$$\frac{Y(S)}{R(S)} = \frac{12000}{S^3+130S^2+1200S+12000}$$

Roots

$S_{1,2} = -4.55 \pm j\,8.86 \quad S_3 = -120.9$

All roots are in LHS => system stable ★

S_3 of -120.9 is >> 10 ($S_{1,2}$) so ignore S portion

we then get:

$$\frac{Y(S)}{R(S)} = \frac{12000}{(S+(4.55+j8.86))(S+(4.55-j8.86))(120.9)}$$

$$\frac{Y(S)}{R(S)} = \frac{99.26}{(S+(4.55+j8.86))(S+(4.55-j8.86))}$$

$$S = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$$

- Find P.O., T_s ($\delta=2\%$), T_p, and e_{ss} to step & ramp I/P's

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{8.86198166148} = \boxed{0.355\,s}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{4.55246153506} = \boxed{0.879\,s}$$

$\zeta\omega_n = 4.55246153506$

$\omega_n = \dfrac{4.55246153506}{\zeta}$ Next page

Since $\omega_n = \frac{4.55246}{\zeta}$

$\omega_n\sqrt{1-\zeta^2} = 8.86198$

$\frac{4.55246}{\zeta}\sqrt{1-\zeta^2} = 8.86198$

$\left(\zeta \frac{8.86198}{4.55246}\right)^2 = 1-\zeta^2$

$\zeta^2\left(1+\left(\frac{8.86198}{4.55246}\right)^2\right) = 1$

$\zeta = \sqrt{\frac{1}{1+\left(\frac{8.86198}{4.55246}\right)^2}} = \underline{\underline{0.4569}}$

6.0

$P.O. = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} = e^{\left(\frac{-0.4569\pi}{\sqrt{1-(0.4569)^2}}\right)} \times 100$

$\boxed{P.O. = 19.9\%}$

Order of system is 1 so

Step I/P

$\underline{\underline{ess = 0}}$ ✓

$Kp = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{240 \cdot 50}{(s+10)(s+120)(s)} = \underline{\underline{\infty}}$

$ess = \frac{1}{1+\infty} = \underline{\underline{0}}$

Ramp I/P

$ess = 1/Kv$

$Kv = \lim_{s\to 0} \frac{s(240)(50)}{s(s+10)(s+120)} = \frac{240(50)}{(120)(10)}$

$ess = 1/10 = 0.1$

$= 10$

$\boxed{ess = 10\%}$ ✓ fairly good steady state error