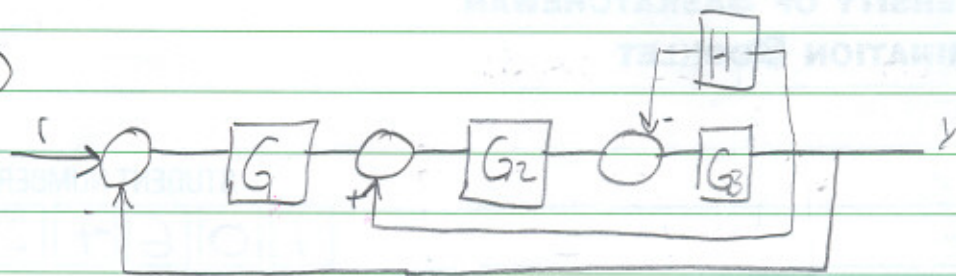


①

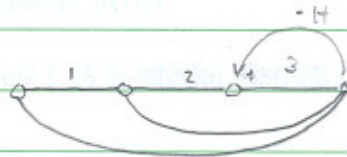


$$P = G_1 G_2 G_3 \quad \frac{Y}{R} = \frac{P}{1 + G_3 H + G_1 G_2 G_3 - G_2 G_3}$$

$$L_1 = -G_3 H$$

$$L_2 = G_2 G_3$$

$$L_3 = -G_1 G_2 G_3$$



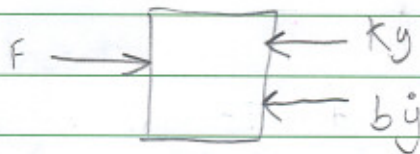
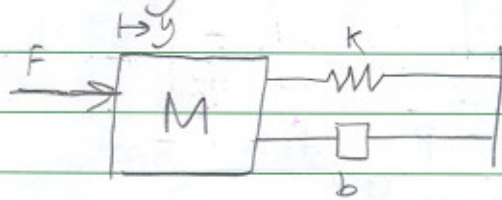
Transfer function:

$$\frac{Y}{R} = \frac{G_1 G_2 G_3}{1 + G_3 H + G_1 G_2 G_3 - G_2 G_3}$$

6.0

P.S. HOPE

② Looking at mass



$$M = 2.4 \text{ kg}$$

$$k = 30 \text{ N/m}$$

$$b = 10 \text{ N s/m}$$

$$F = 1 \text{ N}$$

$$F - ky - bj = M \frac{d^2 y}{dt^2}$$

$$\delta = 2\% = 0.02$$

$$F - kY - bsY = Ms^2 Y$$

$$F = Y(Ms^2 + bs + k)$$

$$Y(s) = \frac{1}{Ms^2 + bs + k}$$

general form

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{Y}{F} = \frac{1/M}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

5.5

Steady state value = $\frac{1}{km} = \frac{1}{7 \frac{\text{kg N}}{\text{m}}}$

$$2\zeta\omega_n = b/m = \zeta\omega_n = \frac{b}{2M} \quad \omega_n = \sqrt{k/m}$$

or $\omega_n = \sqrt{\frac{k}{m}}$

$$T_s = \text{settling time} = \frac{4}{\zeta\omega_n} \text{ for } \delta = 2\%$$

$$T_s = \frac{4(2M)}{b} \Rightarrow T_s = 1.92 \text{ seconds}$$



$$\text{Transfer Function} = \frac{P}{1+P}$$

to be stable:

- ✓ - no zero-pole cancellations
- no ^{pole} roots in the right-hand plane

$$(a) P(s) = \frac{161}{(s+5)(s+4)}$$

$$T(s) = \frac{161}{(s+5)(s+4) + 161}$$

$$\text{den} = s^2 + 9s + (20 + 161) = s^2 + 9s + 181$$

$$\text{roots} = \frac{-9 \pm \sqrt{81 - 4(181)}}{2}$$

$$\text{roots} = -4.5 \pm \sqrt{-160.75}$$

all roots in the left hand side of s-plane \Rightarrow STABLE

Unit Step Input

$$K_p = \lim_{s \rightarrow 0} P(s) = \frac{161}{20} = \underline{8.05}$$

$$e_{ss} = \frac{1}{1+K_p} = 0.1105$$

Error at steady state $e_{(t)}$ = 11.05%

Ramp Input.

$$K_v = \lim_{s \rightarrow 0} s P(s) = \lim_{s \rightarrow 0} \frac{s \cdot 161}{(s+5)(s+4)} = 0$$

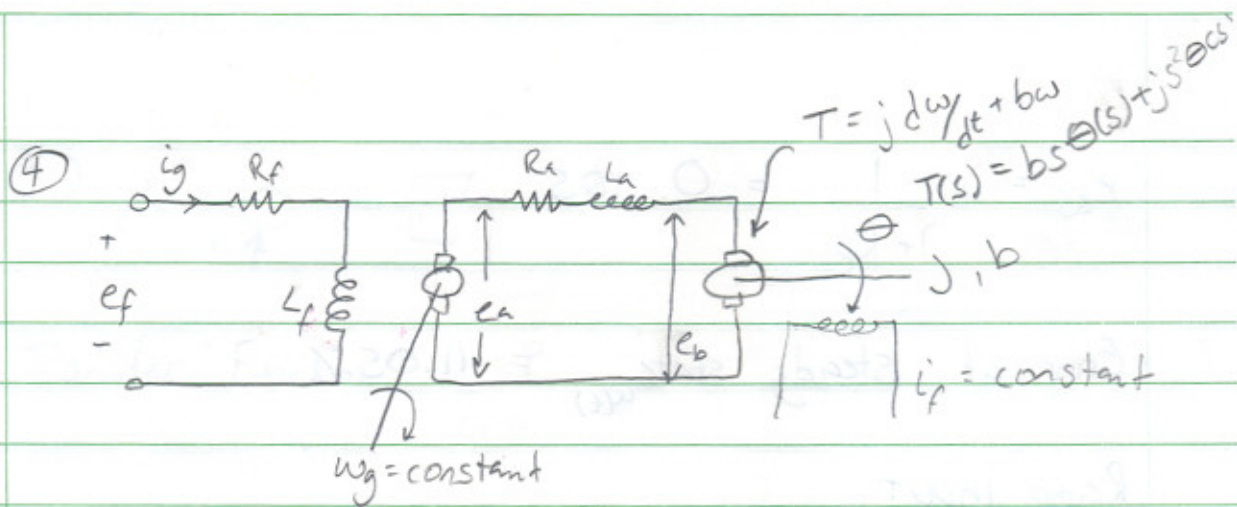
$P(s)$ is type 0, so this is right.
The error signal = $\frac{1}{K_v} = \infty$.

$$s^2 + 9s + 181$$

s^2	1	181	}	$\frac{0 - 181(9)}{-9} = 181$
s	9	0		
1	181			

no sign changes \Rightarrow STABLE

part b?



$e_a = K_g i_g$ find $\frac{\Theta}{e_f}$

$P_{elec} = e_a i_a$
 $P_{mech} = T \omega_g$

$e_a i_a = T \omega_g$

$T = \frac{e_a i_a}{\omega_g} = \frac{K_g i_g i_a}{\omega_g}$

$K_t = \frac{1}{\omega_g}$

$T = K_t K_g i_g i_a$

$e_a - e_b = i_a R_a + L \frac{di_a}{dt}$

$e_f = i_g R_f + L_f \frac{di_g}{dt}$

$E_a - E_b = I_a R_a + L_s \frac{dI_a}{dt}$

$E_a - E_b = I_a (R_a + L_s)$ ✓

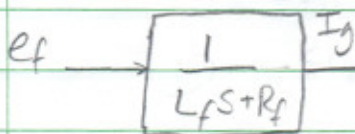
$e_f(s) = I_g(s) R_f + L_f s I_g(s)$

$I_a = \frac{E_a - E_b}{R_a + L_s}$

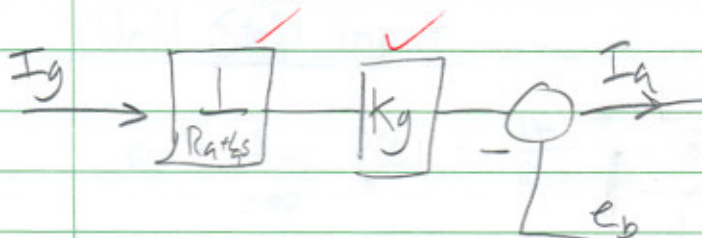
$E_f(s) = I_g(s) (L_f s + R_f)$

$E_a = K_g I_g$

$I_g = \frac{E_f(s)}{L_f s + R_f}$



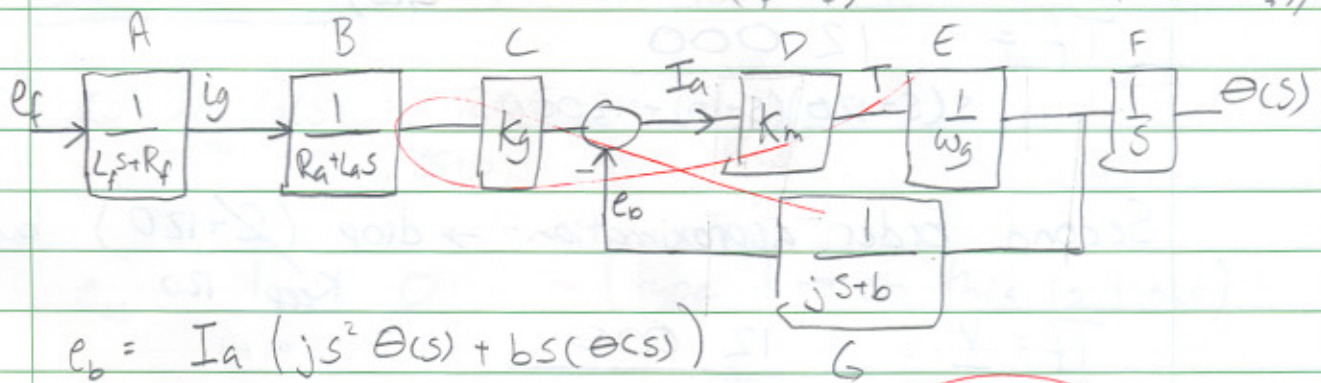
$I_a = \frac{K_g I_g - E_b}{R_a + L_s}$



$$i_a = k_2 k i_f = k_m$$

$$\Theta(s) = \frac{T(s)}{(s^2 + bs)} = \frac{T(s)}{s(s+b)}$$

$$T(s) = k_1 k_g I_g I_a = k_1 k_g \frac{E_f}{(L_f s + R_f)} \left(\frac{-E_b}{R_a + L_a s} + \frac{k_g E_f}{(L_g s + R_g)(R_g + L_g s)} \right)$$



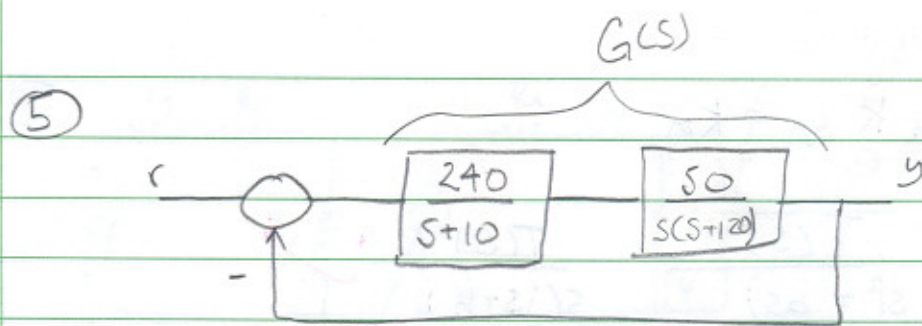
$$e_b = I_a (j s^2 \Theta(s) + b s (\Theta(s)))$$

3.5

$$T = k k_f i_f i_a = k_m i_a$$

$$\Theta(s) = \frac{A B C D E F}{E_f(s) (1 + D E G)} = \frac{A B C F D}{\frac{1}{E G} + D}$$

~~$$\Theta(s) = \frac{k_g k_m}{s(L_f s + R_f)(R_a + L_a s) \omega_g [(L_g s + R_g)(R_g + L_g s) + k_m]}$$~~



Real Transfer function = $\frac{G(s)}{1+G(s)}$

$$Tf = \frac{12000}{s(s+120)(s+10) + 12000}$$

Second order approximation: \rightarrow drop $(s+120)$ but keep 120

$$Tf = \frac{y}{R} = \frac{12000}{s(s+10)(120) + 12000}$$

$$y/R = \frac{12000}{120s^2 + 1200s + 12000}$$

$$y/R = \frac{100}{s^2 + 10s + 100}$$

$$2\zeta\omega_n = 10$$

$$\omega_n^2 = 100 \Rightarrow \omega_n = 10$$

$$\zeta\omega_n = 5$$

$$\zeta = 0.5$$

$$PO = \exp\left(\frac{-0.5\pi}{(1-0.5^2)^{1/2}}\right) \Rightarrow PO = 0.1630$$

$$\text{Percent Overshoot} = 16.30\%$$

$$T_s = \frac{4}{3\zeta\omega_n} \quad (\zeta = 2\%) \Rightarrow T_s = 0.8 \text{ seconds}$$

$$T_p = \frac{\pi}{\omega_n(1-\zeta^2)^{1/2}} \Rightarrow T_p = 0.3628 \text{ seconds}$$

Step Input:

(all roots in LHP $\rightarrow -5 \pm 8.66j$)

$$k_p = \lim_{s \rightarrow 0} G(s) = \frac{12000}{(s+10)s(120)} = \infty$$

$$e_{ss} = \frac{1}{1+\infty} = 0 \quad \checkmark \text{ (type 1 } \rightarrow \text{ so this is right)}$$

ramp input

6.0

$$k_v = \lim_{s \rightarrow 0} sG(s) = \frac{s \cdot 12000}{(s+10)s(120)} = \frac{12000}{1200} = 10 \quad \checkmark$$

$$e_{ss} = \frac{1}{k_v} = \frac{1}{10} = 0.1 = 10\% \quad \checkmark$$