University of Saskatchewan
EE 480.3 Digital Control Systems
Midterm Exam. February 10, 2009

Note: 2 hour open-book exam. You must sign the tally sheet when you submit your exam booklet(s) to the instructor.

Instructor: K. Takaya

1. (20) An exponential function $f(t) = e^{-bt}$ is sampled every $T = 0.2$ second, i.e. $t = kT$ for $k = 0, 1, 2, \cdots$.

1. Find the z-transform $F(z)$.
2. The $F(z)$ happened to be

$$F(z) = 1 + \frac{1}{2}z^{-1} + (\frac{1}{2})^2 z^{-2} + (\frac{1}{2})^3 z^{-3} + \cdots$$

What is the value of b. Show the process of your calculations.

2. (30) Find the control canonical state-space formulation (state equations) for a discrete time system described by the difference equation,

$$y(k+2) - 0.3y(k+1) + 0.02y(k) = e(k+2) - 1.7e(k+1) + 0.72e(k)$$

where, y is output and e is input.

1. Write the transfer function $\dfrac{Y(z)}{E(z)}$.
2. Draw a simulation diagram of this system in the control canonical form.
3. Defining the output of the first delay element as a state x_2 and the output of the second delay element as a state x_1 (bottom up order), write a set of state equations, and an output equation.
4. List the poles and zeros of this system.

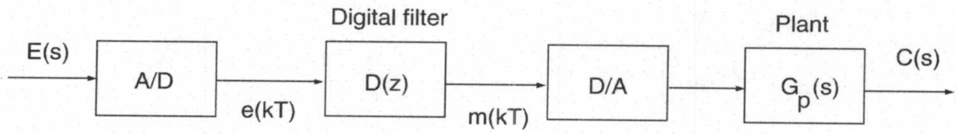
3. (25) For the system shown in the Figure below, the digital filter solves the difference equation

$$m(k) = 0.8m(k-1) + 0.2e(k).$$

The plant transfer function $G_p(s)$ is given by

$$G_p(s) = \frac{s}{(s+1)(s+2)}$$

The sampling rate is 10 Hz or $T = 0.1$ sec. The Digital to Analog converter (D/A) is a Zero Order Hold (ZOH).



1. Write the transfer function of the digital filter, $D(z)$
2. Find the system transfer function $\frac{C(z)}{E(z)}$. You must show the entire mathematical process to calculate the starred transform, then to convert to the z-transform.

4. (25) There is a second order system whose damping ratio is $\zeta = 0.8660$ and natural frequency is $\omega_n = 2$ rad/s. Also, the DC gain is known to be 1. Assume that the system has only two poles but no zeros.

1. Write the transfer function $G(s)$ in the Laplace transform.
2. Sketch the root locus of this system.
3. The system $G(s)$ and a variable gaim K is combined to make an open loop system. This open loop system $K\,G(s)$ is then closed with the unity gain feedback. When the gain K is set $K = 2$, where are the closed-loop poles? Calculate the closed loop poles, then mark those on the root locus drawn in the previous question.

— The End —