

22.8

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EE351.3 Mid-term test, Signals and Systems

7 Questions. Duration 90 minutes.

You may use a formula sheet.

Attempt all the questions. Justify your answers.

[4 marks] 1) Take the discrete-time signal $x(n] = \sin(n\pi/2)(u(n) - u(n - 4))$, where $u(n)$ is the unit step function. Sketch $x(n)$.

$$x(n) = 0 \text{ for } n < 0$$

$$x(0) = \sin(0) [u(0) - u(-4)] = 0$$

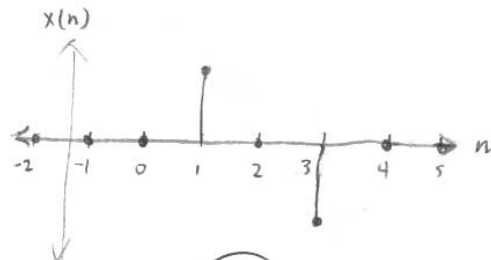
$$x(1) = \sin\left(\frac{\pi}{2}\right) [u(1) - u(-3)] = 1$$

$$x(2) = \sin(\pi) [u(2) - u(-2)] = 0$$

$$x(3) = \sin\left(\frac{3\pi}{2}\right) [u(3) - u(-1)] = -1$$

$$x(4) = \sin(2\pi) [u(4) - u(0)] = 0$$

$$x(n) = 0 \text{ for } n > 4$$



4.0

[2 marks] 2) For what values of ω is the signal $x(n) = e^{j\omega n}$ periodic?

$$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$$

assuming a period T of 5 for $e^{j\omega n} = \cos(\omega n) + j\sin(\omega n)$

$$\omega = \frac{2\pi}{5}$$

of n intervals

1.0

[5 marks] 3) Take a system with an impulse response $h(n) = \delta(n) - \delta(n-1)$. Find the response of the system to the input $x(n) = \delta(n) - \delta(n-1) + \delta(n-2)$.

Both $h(n), x(n) = 0$ for $n < 0$

$$h(0) = \delta(0) - \delta(-1) = 1$$

$$h(1) = \delta(1) - \delta(0) = -1$$

$$h(n) = 0 \text{ for } n > 1$$

$$h(n) = [1 \ -1 \ 0 \ 0 \ 0]$$

$$x(0) = \delta(0) - \delta(-1) + \delta(-2) = 1$$

$$x(1) = \delta(1) - \delta(0) + \delta(-1) = -1$$

$$x(2) = \delta(2) - \delta(1) + \delta(0) = 1$$

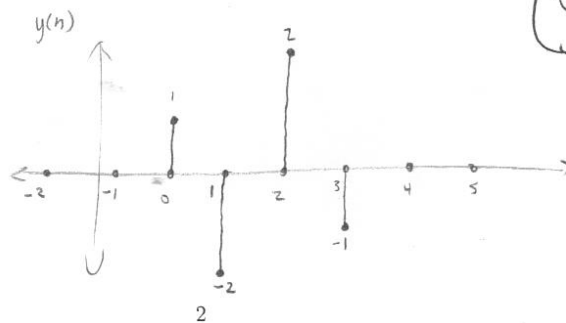
$$x(n) = 0 \text{ for } n > 2$$

$$x(n) = [1 \ -1 \ 1 \ 0 \ 0]$$

$$y(n) = x(n) * h(n)$$

$$y(n) = [1 \ -2 \ 2 \ -1 \ 0]$$

| | | | | | |
|----|----|----|----|---|---|
| | 1 | -1 | 1 | 0 | 0 |
| 1 | 1 | -1 | 1 | 0 | 0 |
| -1 | -1 | 1 | -1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |



8.5

[9 marks] 4) A discrete-time system has the relationship

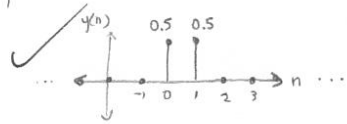
$$y(n] = 0.5x(n) + 0.5x(n-1)$$

between its input $x(n)$ and its output $y(n)$.

a) Determine $y(n)$ if the input is the $\delta(n)$. Sketch the output.

$$\begin{aligned} x(n] &= \delta(n) \\ y(n] &= 0.5\delta(n) + 0.5\delta(n-1) \\ y(n] &= 0 \text{ for } n < 0 \\ y(0] &= 0.5\delta(0) + 0.5\delta(-1) = 0.5 \\ y(1] &= 0.5\delta(1) + 0.5\delta(0) = 0.5 \end{aligned}$$

$$y(n] = 0 \text{ for } n > 1$$



b) Is this system LTI?
linear? **YES** because $y(n+a] = 0.5x(n+a] + 0.5x(n+a-1]$

Time Invariant? **YES** because the function $y(n)$ does not depend on continuous time t .

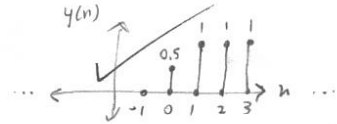
c) Is this system stable? Is it memoryless?

Stable? **YES** $\sum_{n=-\infty}^{\infty} |y(n)] = 1$ this value does not run in to $\pm\infty$ so the system has to be stable.

Memoryless? **NO** because $y(n] = 0.5x(n] + 0.5x(n-1]$ $y(n]$ calls on a previous value of n

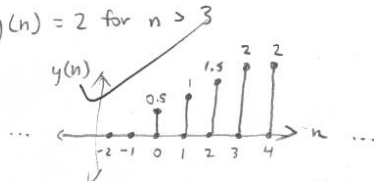
d) Determine $y(n)$ when $x(n)$ is a unit step function $u(n)$?

$$\begin{aligned} x(n] &= u(n) \\ y(n] &= 0.5u(n) + 0.5u(n-1) \\ y(n] &= 0 \text{ for } n < 0 \\ y(0] &= 0.5u(0) + 0.5u(-1) = 0.5 \\ y(1] &= 0.5u(1) + 0.5u(0) = 1 \\ y(n] &= 1 \text{ for } n > 1 \end{aligned}$$



e) What is the output if the input is $x(n) = \delta(n) + \delta(n-1) + 2u(n-2)$. Sketch $y(n)$

$$\begin{aligned} y(n] &= 0.5[\delta(n) + \delta(n-1) + 2u(n-2)] + 0.5[\delta(n-1) + \delta(n-1-1) + 2u(n-2-1)] \\ y(n] &= 0.5\delta(n) + 0.5\delta(n-1) + u(n-2) + 0.5\delta(n-1) + 0.5\delta(n-2) + u(n-3) \\ y(n] &= 0.5\delta(n) + \delta(n-1) + 0.5\delta(n-2) + u(n-2) + u(n-3) \\ y(n] &= 0 \text{ for } n < 0 \\ y(0] &= 0.5\delta(0) + \delta(-1) + 0.5\delta(-2) + u(-2) + u(-3) = 0.5 \\ y(1] &= 0.5\delta(1) + \delta(0) + 0.5\delta(-1) + u(-1) + u(-2) = 1 \\ y(2] &= 0.5\delta(2) + \delta(1) + 0.5\delta(0) + u(0) + u(-1) = 1.5 \\ y(3] &= 0.5\delta(3) + \delta(2) + 0.5\delta(1) + u(1) + u(0) = 2 \\ y(n] &= 2 \text{ for } n > 3 \end{aligned}$$



f) What is the output of the system if $x(n) = (-1)^n$?

$$\begin{aligned} y(n] &= 0.5(-1)^n + 0.5(-1)^{n-1} \\ y(-2] &= 0.5(-1)^{-2} + 0.5(-1)^{-3} = 0 \\ y(-1] &= 0.5(-1)^{-1} + 0.5(-1)^{-2} = 0 \\ y(0] &= 0.5(-1)^0 + 0.5(-1)^{-1} = 0 \\ y(1] &= 0.5(-1)^1 + 0.5(-1)^0 = 0 \\ y(2] &= 0.5(-1)^2 + 0.5(-1)^1 = 0 \end{aligned}$$

No matter what value n is, the answer will be zero

$$y(n] = 0 \text{ for all } n$$

g) What does this system do?

$y(n)$ finds the derivative value
Average!

[4 marks] 5) Take the signal $x(t) = 1 + 3\sin(3\pi t) + 6\sin(12\pi t)$. Find the period and the exponential Fourier series coefficients of the signal.

$$x(t) = 1 + \frac{3}{2j} e^{j3\pi t} - \frac{3}{2j} e^{-j3\pi t} + \frac{6}{2j} e^{j12\pi t} - \frac{6}{2j} e^{-j12\pi t} \quad \omega = 3\pi$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j\omega_k t}$$

$$x(t) = \dots + c_{-4} e^{j/2\pi t} + c_{-3} e^{j9\pi t} + c_{-2} e^{j6\pi t} + c_{-1} e^{j3\pi t} + c_0 + \dots$$

$$\dots + c_1 e^{-j3\pi t} + c_2 e^{-j6\pi t} + c_3 e^{-j9\pi t} + c_4 e^{-j12\pi t} + \dots$$

$$c_{-4} = \frac{3}{j}$$

$$c_0 = 1$$

$$c_4 = -\frac{3}{j}$$

$$c_{-3} = 0$$

$$c_1 = -\frac{3}{2j}$$

$$c_{-2} = 0$$

$$c_2 = 0$$

$$c_{-1} = \frac{3}{2j}$$

$$c_3 = 0$$

$$T = \frac{2\pi}{\omega} = \frac{2}{3}$$

3.5
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[2 marks] 6) (a) When are the even harmonics of a signal zero?

The even harmonics are zero when the coefficients of that signal satisfy $a_k = -a_k = 0$

(b) When are all exponential Fourier series coefficients of a signal real?

The coefficients of an exponential Fourier series are real when they satisfy the proof

$$a_k = a_{-k}^*$$

where a_k is a coefficient and a_{-k}^* is its conjugate counterpart.

0.0

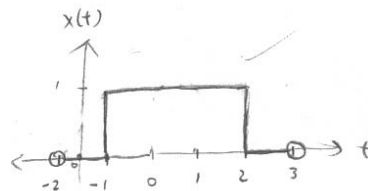
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[4 marks] 7) Take the signal with a period 5 as given below

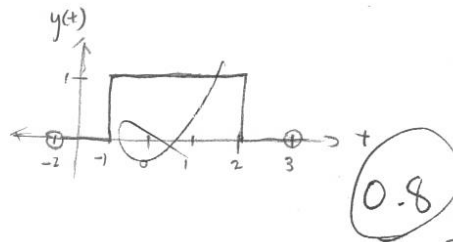
$$x(t) = (u(t+1) - u(t-2)) \text{ for } -2 < t < 3.$$

If $y(t) = \cos(2\pi t)x(t)$. Sketch $x(t)$ and $y(t)$. Find the exponential Fourier series coefficient of $y(t)$.

$$\begin{aligned} x(-1) &= u(0) - u(-3) = 1 \\ x(0) &= u(1) - u(-2) = 1 \\ x(1) &= u(2) - u(-1) = 1 \\ x(2) &= u(3) - u(0) = 0 \end{aligned}$$



$$\begin{aligned} y(-1) &= \cos(-2\pi)(1) = 1 \\ y(0) &= \cos(0)(1) = 1 \\ y(1) &= \cos(2\pi)(1) = 1 \\ y(2) &= \cos(4\pi)(0) = 0 \end{aligned}$$



$$a_0 = \frac{1}{T} \int_0^T y(t) dt = \frac{1}{5} \int_0^5 \cos(2\pi t) dt = \frac{1}{5} [-\sin(2\pi t)]_0^5$$

$$a_0 = 0$$

$$a_k = \frac{2}{T} \int_0^T y(t) \cos(2\pi kt) dt = \frac{2}{5} \int_0^5 \cos(2\pi t) \cos(2\pi kt) dt$$

$$a_{-1} = 1 \quad a_1 = 1 \quad a_2 = 0$$

$$b_k = \frac{2}{T} \int_0^T y(t) \sin(2\pi kt) dt = \frac{2}{5} \int_0^5 \cos(2\pi t) \sin(2\pi kt) dt$$

$$b_{-1} = 0 \quad b_1 = 0 \quad b_2 = 0$$

the coefficients were solved via HP 50g calculator.