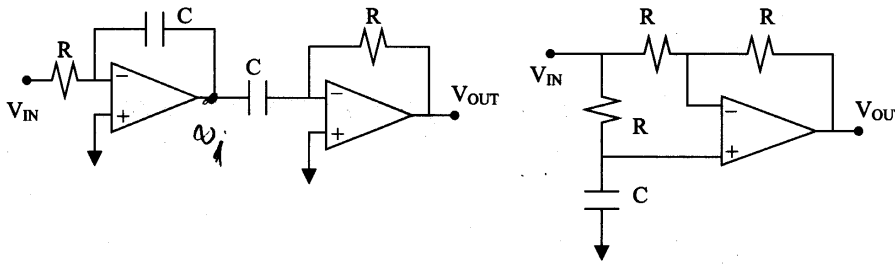


Solution  
Open books, open notes.  
Good luck and have a Merry Christmas.

1. Question 1: (20 marks)

For the circuits (a) and (b) below, derive transfer functions  $V_{OUT}/V_{IN}$  as a function of frequency. For  $R=10K$  and  $C=15.9nF$ , sketch amplitude and phase response of  $V_{OUT}/V_{IN}$ .

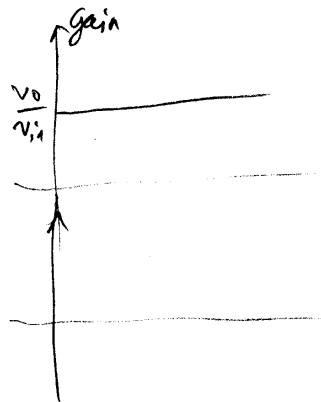


(a)

$$v_1 = -\frac{Z_c}{R} v_{in}, \quad v_o = -\frac{R}{Z_c} v_1$$

$$\Rightarrow v_o = \left(-\frac{R}{Z_c}\right) \left(-\frac{Z_c}{R}\right) v_{in}$$

$$v_o = v_{in} \Rightarrow T(s) = \frac{v_o}{v_i} = 1$$



(b)

$$v_+ = v_- \Rightarrow \frac{Z_c}{Z_c + R} v_{in} = \frac{1}{2} v_o + \frac{1}{2} v_{in} = \frac{1}{2} (v_o + v_{in})$$

$$\frac{1}{2} v_o = -\frac{1}{2} v_{in} + \frac{\frac{1}{sC}}{\frac{1}{sC} + R} v_{in} = v_{in} \left( \frac{1}{1 + j\omega RC} - \frac{1}{2} \right)$$

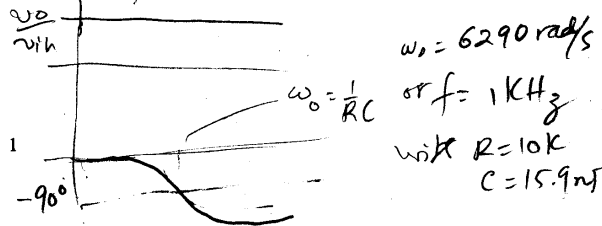
$$v_o = 2 v_{in} \left( \frac{1}{1 + j\omega RC} - \frac{1}{2} \right) = 2 v_{in} \left( \frac{2 - 1 - j\omega RC}{2(1 + j\omega RC)} \right)$$

$$\frac{v_o}{v_{in}} = \frac{1 - j\omega RC}{1 + j\omega RC}$$

$$\phi = \tan^{-1} \frac{-\omega RC}{1} - \tan^{-1} \left( \frac{\omega R}{1} \right) = -2 \tan^{-1}(\omega RC)$$

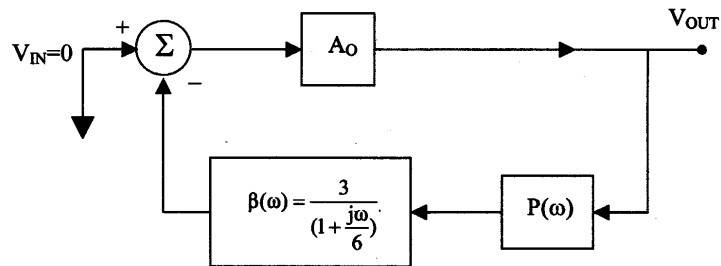
$$\omega = 0 \Rightarrow \frac{v_o}{v_{in}} = 1, \quad \phi = 0$$

$$\omega = \frac{1}{RC} \Rightarrow \frac{v_o}{v_i} = 1, \quad \phi = -2 \tan^{-1}(1) = -90^\circ$$



2. Question 2: (20 marks)

The feedback diagram shown below describes an oscillator circuit. In this case,  $|P(\omega)|=0.1$  and  $\angle P(\omega) = -135^\circ$  for all  $\omega$ .



- Find the frequency of oscillation.
- Find the minimum value of  $A_0$  needed to maintain oscillation.

⊛ to get  $-180^\circ$ ,  $\angle \beta$  must be  $-45^\circ$  since  $\angle P(\omega) = -135^\circ$   
 $-45^\circ = -\tan^{-1}(\frac{\omega}{6})$  or  $\frac{\omega}{6} = 1$   $\omega = 6 \text{ rad/s}$

$$f = \frac{6}{2\pi} = \underline{\underline{0.95 \text{ Hz}}}$$

⊛ Magnitude = 1

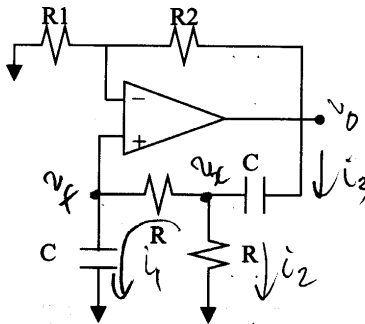
$$A\beta P = 1 \Rightarrow A = \frac{1}{\beta P} = \frac{1}{|B_{\omega}| 0.1}$$

$$|\beta(\omega)| = \frac{3}{|1+j|} = \frac{3}{\sqrt{2}}$$

$$A = \frac{1}{\frac{3}{\sqrt{2}} (0.1)} = \frac{\sqrt{2}}{0.3} = \underline{\underline{4.7}}$$

3. Question 3: (20 marks)

For the circuit below, find the loop gain  $L(s)$ ,  $L(j\omega)$ , the frequency for zero loop-phase. Find  $R_2/R_1$  for oscillation.



Many way to solve:

use KCL at nodes, find  $v_f$  vs  $v_o$

$$\frac{v_f}{v_o} = \frac{1}{sRC + 3 + \frac{1}{sRC}} = \frac{1}{3 + j(\omega RC - \frac{1}{\omega RC})}$$

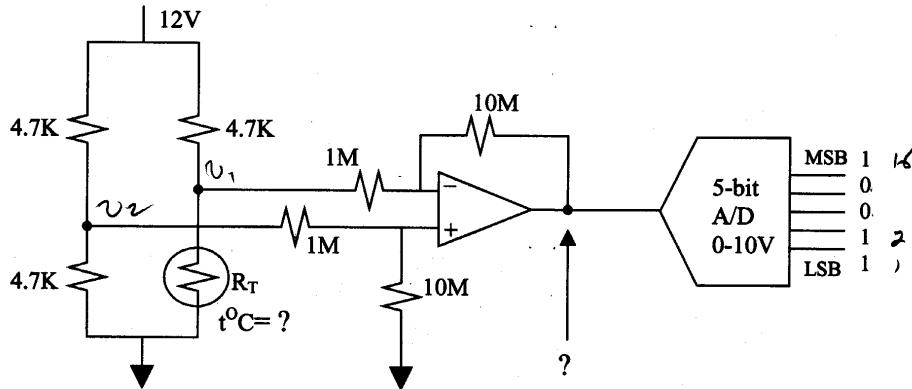
phase = 0  $\Rightarrow \omega RC = \frac{1}{\omega RC}$  or  $\boxed{\omega = \frac{1}{RC}}$

then at  $\omega = \frac{1}{RC}$ ,  $\frac{v_f}{v_o} = \frac{1}{3}$

or  $\frac{v_o}{v_f} = 3 = 1 + \frac{R_2}{R_1}$  or  $\boxed{\frac{R_2}{R_1} = 2}$

4. Question 4: (20 marks)

Consider the circuit in a temperature measurement below. The A/D is a 5-bit successive-approximation A/D converter type with an analog span of 0 to 10V, find the input voltage of the A/D converter. The thermistor,  $R_T$ , has a resistance of 2K at 20°C and the coefficient  $\beta$  is assumed to be constant at 3650, find temperature of the thermistor.



A/D 5bit  $\rightarrow 2^5 = 32$  steps or  $\frac{10}{2^5} = 0.3125V$  each step.

Total 19 steps  $\Rightarrow$  input =  $(19)(0.3125V) = 5.9375V$

The amplifier has a gain of:

$$v_o = -10v_1 + \frac{10}{11}v_2(1+10) = 10(v_2 - v_1)$$

$$\therefore v_2 - v_1 = \frac{5.9375V}{10} = 0.59375V$$

$$v_1 = v_2 - 0.59375V = 6V - 0.59375V = 5.40625V$$

$$v_1 = v_{RT} = 5.40625V = \frac{R_T}{R_T + 4K} \cdot 12V \Rightarrow R_T = 3.85K\Omega$$

$$R_T = R_0 e^{\beta(\frac{1}{T} - \frac{1}{T_0})}$$

$$e^{\beta(\frac{1}{T} - \frac{1}{T_0})} = \frac{R_T}{R_0} = 1.9267 \Rightarrow \beta(\frac{1}{T} - \frac{1}{T_0}) = \ln(1.9267)$$

$$\frac{1}{T} = \frac{\ln(1.9267)}{\beta} + \frac{1}{T_0}$$

$$T = 19.9^\circ C$$

5. Question 5: (20 marks)

In a digital instrumentation system to measure velocity of a fluid pipe, the A/D converter has a sampling rate of 20Ksample/second. Find the Nyquist frequency of the analog signal from the transducer. Design an active filter for anti-aliasing purpose in front of the A/D converter. The filter should have a cut off frequency at Nyquist frequency with a selection of  $F_{50}/F_3$  is at least 3. Since the output signal of the transducer has a wide range of frequency, no ripple is allowed in the filter passband and only 10K resistors are available to realize the filter.

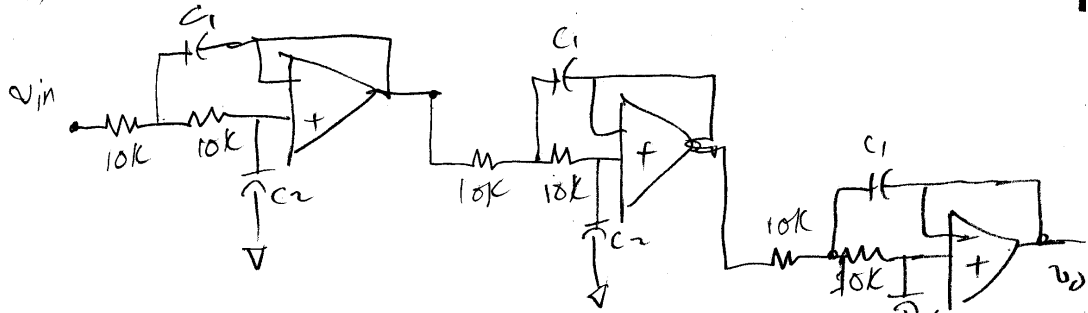
Table 12-1. Design Data for Chebyshev Filters

Ripple = 0 dB (Butterworth)		Cutoff frequency = section frequency = 1.0								
Number of sections	$\frac{F_{50}}{F_3}$	Q Sect 1	Q Sect 2	Q Sect 3	Q Sect 4	Q Sect 5	Q Sect 6	Q Sect 7	Q Sect 8	
	1	17.79	0.7071							
2	4.22	0.5411	1.305							
3	2.61	0.5176	0.7071	1.932						
4	2.05	0.5098	0.6014	0.8999	2.563					
5	1.78	0.5062	0.5012	0.7071	1.101	3.198				
6	1.61	0.5043	0.5412	0.6902	0.8213	1.307	3.831			
7	1.51	0.5032	0.5237	0.5905	0.7071	0.9401	1.514	4.468		
8	1.43	0.5024	0.5225	0.5889	0.5486	0.7882	1.081	1.722	5.101	

Ripple = 0.1 dB		Cutoff frequency = 1.0									
Number of sections	$\frac{F_{50}}{F_3}$	F Sect 1	F Sect 2	F Sect 3	F Sect 4	F Sect 5	F Sect 6	F Sect 7	F Sect 8		
		Q Sect 1	Q Sect 2	Q Sect 3	Q Sect 4	Q Sect 5	Q Sect 6	Q Sect 7	Q Sect 8		
1	18.59	0.9221									
2	3.36	0.7674	0.9491								
3	1.96	0.6180	2.185								
4	1.52	0.4688	0.7626	0.8717							
5	1.32	0.3923	1.333	4.530							
6	1.22	0.3223	0.6129	0.8483	0.9628						
7	1.16	0.2940	1.184	2.455	3.092						
8	1.12	0.2640	0.8065	0.7282	0.5984	0.9967					
9	1.22	0.2469	1.128	2.045	3.826	12.54					
10	1.16	0.2126	0.4296	0.6314	0.8038	0.9275	0.9920				
11	1.12	0.1866	1.100	1.863	3.123	5.733	17.38				
12	1.12	0.1686	0.3723	0.5639	0.7187	0.8823	0.9459	0.9941			
13	1.12	0.1586	1.084	1.728	2.794	4.403	7.871	24.40			
14	1.12	0.1486	0.3380	0.4620	0.6463	0.7796	0.8882	0.9882	0.9965		
15	1.12	0.1375	1.074	1.748	2.619	3.860	5.883	10.34	34.82		

$$f_N = \frac{f_s}{2} = \frac{20\text{kHz}}{2} = 10\text{kHz}$$

$$\frac{f_{50}}{f_3} = 3 \Rightarrow 3 \text{ section Using Sallen Key (Ripple = 0dB)}$$



$$f_1 = 10\text{kHz}$$

$$Q_1 = 0.5176$$

$$f_2 = 10\text{kHz}$$

$$Q_2 = 0.7071$$

$$f_3 = 10\text{kHz}$$

$$Q_3 = 1.932$$

$$C_1 = \frac{Q}{\pi f R}$$

$$C_2 = \frac{C_1}{4Q^2}$$

$$C_1 = 1647\text{nF}$$

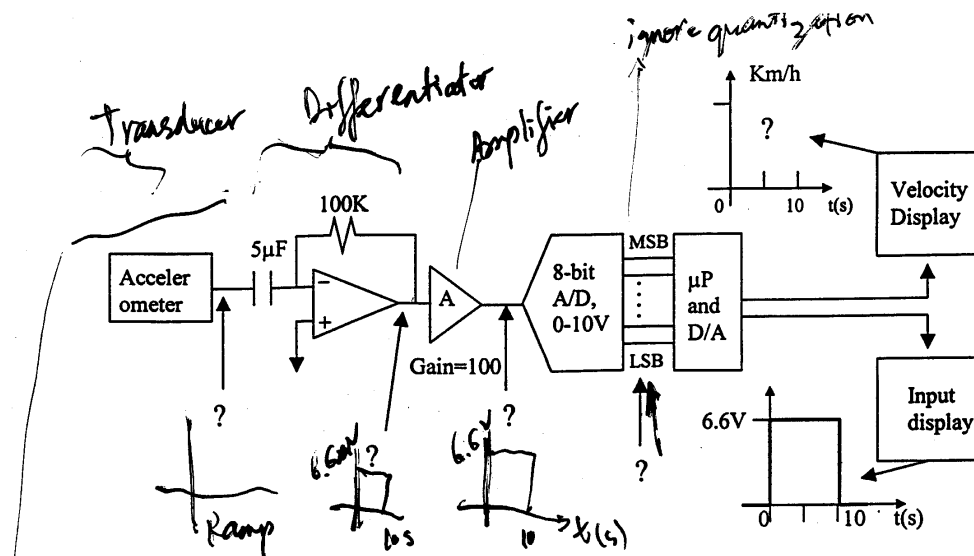
$$C_2 = 1.537\text{nF}$$

$$C_1 = 2.251\text{nF}$$

$$C_2 = 1.125\text{nF}$$

$$C_1 = 6.1497\text{nF}$$

$$C_2 = 0.419\text{nF}$$

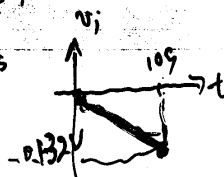


The above arrangement is used to measure velocity of a vehicle (not a good design). The waveform shown at the input display is the output of the D/A converter (data from A/D connects directly to D/A). Ignore quantization error, find the A/D output word. Sketch analog input voltage waveform at the A/D converter, the amplifier A input and the accelerometer output. The accelerometer has an inversion factor of  $0.25V/m/s^2$  (i.e.,  $250mV$  corresponds to  $1m/s^2$ ), find final velocity of the vehicle if its initial velocity is  $100Km/h$  and sketch the vehicle velocity.

$$v_0 = -\frac{1}{RC} \frac{dv_i}{dt} \quad \text{or} \quad v_i = -\int RC \, dv_0$$

$$v_i = \frac{1}{(100K)(5\mu F)} \int_0^{10s} (6.6mV) dt$$

$$= -2 (0.066t) \Big|_0^{10s}$$



$$v = at + v_0$$

$$= (-0.132)(10s) + \frac{100K}{hr}$$

convert into  $m/s^2$

$$= 0.526 \frac{m}{s^2}$$

$$\frac{10}{24} = \frac{10}{24} \text{ steps} = 0.0391 \frac{V}{\text{step}}$$

$$\frac{6.6V}{0.0391 \frac{V}{\text{step}}} = 169 \text{ steps}$$

45A      25B  
10101001

