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University of Saskatchewan
Department of Electrical and Computer Engineering

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EE301 Electricity, Magnetism and Fields
Final Examination
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Welcome to the EE301 final examination. This is a closed book and closed notes examination. A formulae sheet is attached. You may use a calculator. The examination lasts **3** hours.

Answer any **6** of the **7** problems. Do not answer more than 6 problems or severe penalties will apply.

Each problem is worth the same; subparts are weighted as indicated. Show your work and briefly explain what you are doing if appropriate; credit will be given only if the steps leading to the answer are clearly shown. If a symmetry argument is used, it is sufficient to write "By symmetry we know that...". Partial credit will be given for partially correct answers if the method used is correct. Be reasonably neat; credit will not be given for illegible answers.

None of the problems require intricate mathematical manipulations. If you get stuck with an impossible integral or equation, you are likely approaching the problem incorrectly.

1. Electrostatics

a) (2) Two point charges of 20 nC and -10 nC are separated by 1 cm. What is the electric field vector at the midpoint between the two charges? What is the total electric flux passing through a spherical surface containing both charges.

b) (8) Space is filled with a charge density ρ_V that varies with the distance from the origin according to the formula (spherical coordinates)

$$\rho_V = \rho_0 \frac{a^2}{r^2} e^{-r/a}$$

where ρ_0 and a are constants. Use Gauss's law to find the electric field produced by this charge.

2. Ampere's law

A current I flows through a straight, infinitely long metal wire with radius a. The current density is uniform inside the wire. Use Ampere's law to determine the magnetic field both inside and outside the wire.

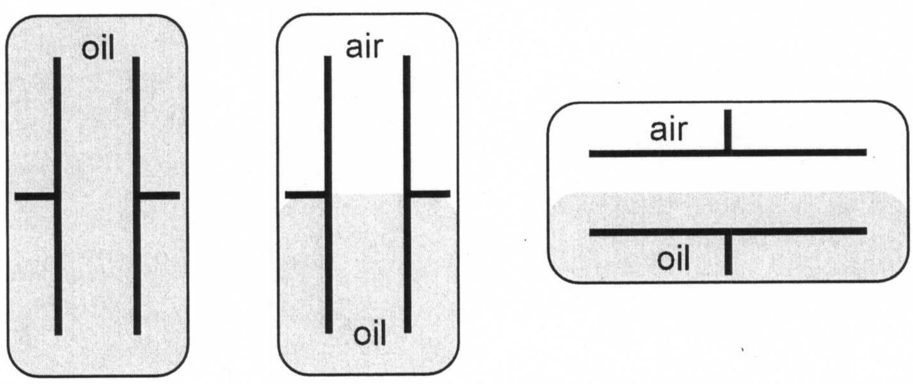
3. Capacitance

A high-voltage capacitor consists of a parallel-plate capacitor sealed in a container that is filled with oil (diagram *a*). The oil has a dielectric constant $\epsilon_R = 2.3$ and a dielectric strength of 10^7 V/m (the dielectric strength is the magnitude of the electric field at which electric breakdown, i.e. a spark, occurs). The plates have an area of 2 m^2 and a separation of 1 cm.

a) (2) Calculate the capacitance and the absolute maximum voltage rating (that is the applied voltage at which breakdown occurs).

b) (4) Because of sloppy maintenance, half the oil is allowed to drain from the capacitor as shown in diagram *b*. Calculate the capacitance and the absolute maximum voltage rating. Note that the dielectric strength of air is 3×10^6 V/m.

c) (4) Incredibly, a forklift backs up into the capacitor and knocks it on its side. The oil now fills half the space between the plates as in diagram *c*. Calculate the capacitance and the absolute maximum voltage rating.



(a) (b) (c)

4. EM waves

A coaxial cable has the dimensions: inner wire radius a and outer shield radius b. A sinusoidal signal is carried by the coaxial cable. The signal produces an electromagnetic wave inside the coaxial cable that is confined to the space between the inner wire and the shield. The solution of the wave equation shows that the electric field has the form (cylindrical coordinates)

$$\vec{E} = E_{\max}\frac{a}{\rho}\,e^{j(\omega t - kz)}\vec{a}_\rho \qquad a \le \rho \le b$$

where $E_{\max}$ is a constant equal to the maximum electric field magnitude in the cable.

a) (1) Is the electromagnetic wave a plane wave? Explain why or why not.

b) (3) Determine the equation for the magnetic field from the appropriate Maxwell equation.

c) (3) Calculate the time-averaged Poynting vector.

d) (3) Determine the equation for the average total power carried by the coaxial cable.

Note that in cylindrical coordinates, the curl is

$$\vec{\nabla} \times \vec{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\vec{a}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\vec{a}_\phi + \frac{1}{\rho}\left(\frac{\partial \rho A_\phi}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)\vec{a}_z$$

and

$$\vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z \qquad \vec{a}_z \times \vec{a}_\rho = \vec{a}_\phi \qquad \vec{a}_\phi \times \vec{a}_z = \vec{a}_\rho$$