November 2, 2006

University of Saskatchewan
Department of Electrical Engineering

EE301 Electricity, Magnetism and Fields
Midterm Examination
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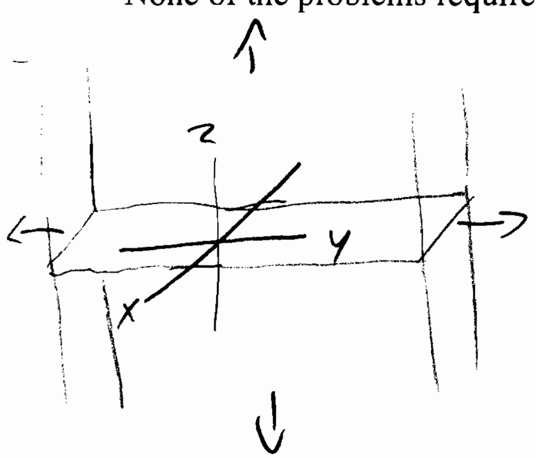
Welcome to the EE301 Midterm. This is a closed book and closed notes examination. A formulae sheet is attached. You may use a calculator. The examination lasts **2** hours.

Each problem is worth the same; subparts of a problem might be weighted unequally. Show your work; credit will be given only if the steps leading to the answer are **clearly** shown. If a symmetry argument is used, it is sufficient to write "By symmetry we know that...". Partial credit will be given for partially correct answers. Be reasonably neat; credit will not be given for illegible answers.

You are to answer **4** of the 5 problems. Do not solve more than four problems or severe penalties will apply.

None of the problems require intricate mathematical manipulations.



Problem 1

a) A charge of 1 nC is located at the point (1, 0) and a charge of -1 nC is located at the point (0, 1). What is the electric field vector and the potential at the origin (0, 0). The points are Cartesian (x, y) and the distances are measured in meters.

b) A charge Q is located at the center of a cube with side a. What is the total electric flux that passes through the walls of the cube?

Problem 2

A slab of charge with width $2a$ has a volume charge density that varies linearly with x,

$$\rho_V = \rho_0 x / a \qquad \text{for } -a < x < a .$$

Note that the volume charge density is negative for $-a < x < 0$ and positive for $0 < x < a$. The volume charge density is zero outside the slab. The slab extends infinitely far in the y and z directions. Use Gauss's law to determine the electric field everywhere.

Problem 3

A capacitor is constructed from two concentric, spherical metal shells. The inner shell has radius a and the outer shell has radius b. the space between the shells is filled with a dielectric with relative permittivity ε_R. Determine a formula for the capacitance.

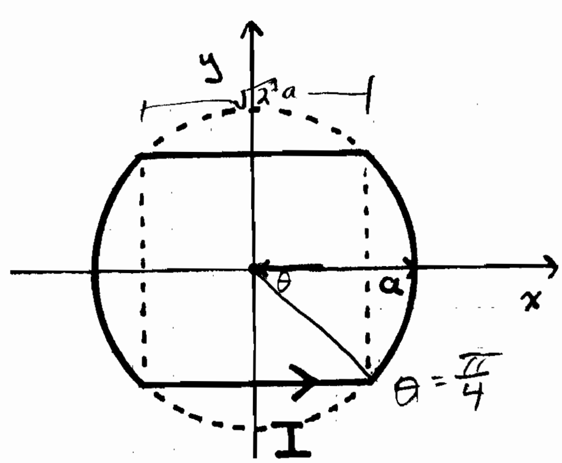
Problem 4

A spherical ball of charge has uniform volume charge density ρ_V and radius a. Determine the total energy in the electric field.

Problem 5

A piece of wire is bent into the shape shown. The arcs are part of a circle of radius a and the straight segments are part of a square of side $\sqrt{2}a$. Each arc subtends an angle of 90°. A current I flows through the wire. Use the law of Biot-Savart to calculate the magnetic field vector at the center. You may be interested to know that

$$\int \frac{dx}{(x^2+b)^{3/2}} = \frac{x}{b(x^2+b)^{1/2}}.$$



Symbols and Constants

F	force	V	electric potential
Q	charge	$\vec{A}$	vector potential
$\vec{E}$	electric field	ρ_V	volume charge density
$\vec{D}$	displacement field	I	current
$\vec{P}$	polarization field	$\vec{j}$	current density
$\vec{H}$	magnetic field	ε_R	relative permittivity
$\vec{B}$	magnetic flux density field	μ_R	relative permeability
$\vec{M}$	magnetization field	$\varepsilon_0 \approx 8.85\times10^{-12}$ F/m	
		$\mu_0 = 4\pi\times10^{-7}$ N/A^2	

Vector Calculus

cross products

Cartesian $\quad \vec{a}_x \times \vec{a}_y = \vec{a}_z \quad \vec{a}_y \times \vec{a}_z = \vec{a}_x \quad \vec{a}_z \times \vec{a}_x = \vec{a}_y$

cylindrical $\quad \vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z \quad \vec{a}_\phi \times \vec{a}_z = \vec{a}_\rho \quad \vec{a}_z \times \vec{a}_\rho = \vec{a}_\phi$

spherical $\quad \vec{a}_r \times \vec{a}_\theta = \vec{a}_\phi \quad \vec{a}_\theta \times \vec{a}_\phi = \vec{a}_r \quad \vec{a}_\phi \times \vec{a}_r = \vec{a}_\theta$

spheres

$$Vol = (4/3)\pi a^3 \quad Area = 4\pi a^2$$

$$dV = r^2 \sin\theta\, dr d\theta d\varphi \quad dS = r^2 \sin\theta\, d\theta d\varphi$$

cylinders $\quad Vol = \pi a^2 L \quad dV = \rho\, d\rho d\varphi dz$

Electrostatics

Coulomb's law

$$\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\vec{a}_{12}$$

point charge $\quad$ field: $\vec{E} = \dfrac{Q\vec{a}_r}{4\pi\varepsilon_0 r^2}$ $\qquad$ potential: $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

charge distribution

$$\vec{E} = \int_V \frac{\rho_V(\vec{r}')}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|^2}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}dV' \qquad V = \int_V \frac{\rho_V(\vec{r}')dV'}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|}$$

Gauss's law

$$\varepsilon_0 \oint_S \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}^{\text{total}} = \int_V \rho_V\, dV$$

$$\oint_S \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}^{\text{free}} = \int_V \rho_V^{\text{free}}\, dV$$

$$\oint_S \vec{P}\cdot d\vec{S} = -Q_{\text{enclosed}}^{\text{bound}} = -\int_V \rho_V^{\text{bound}}\, dV$$

relating $\vec{E}$ and V

$$\vec{E} = -\vec{\nabla}V \qquad V_{AB} = -\int_B^A \vec{E}\cdot d\vec{l}$$