

① 10nC @ (0,0,0)

-5nC @ (1,0,0)

-5nC @ (1,1,1)

Find \vec{E} at (0,1,1)

$$\vec{r}_0 = \vec{a}_y + \vec{a}_z$$

10nC $\vec{r}' = 0$ $\vec{r} - \vec{r}' = \vec{a}_y + \vec{a}_z$ $|\vec{r} - \vec{r}'| = \sqrt{2}$

$$\vec{E} = \frac{10nC}{4\pi\epsilon_0} \left(\frac{\vec{a}_y + \vec{a}_z}{2^{3/2}} \right)$$

-5nC $\vec{r}' = \vec{a}_x$ $\vec{r} - \vec{r}' = -\vec{a}_x + \vec{a}_y + \vec{a}_z$ $|\vec{r} - \vec{r}'| = \sqrt{3}$

$$\vec{E} = \frac{-5nC}{4\pi\epsilon_0} \left(\frac{-\vec{a}_x + \vec{a}_y + \vec{a}_z}{3^{3/2}} \right)$$

-5nC $\vec{r}' = \vec{a}_x + \vec{a}_y + \vec{a}_z$ $\vec{r} - \vec{r}' = -\vec{a}_x$ $|\vec{r} - \vec{r}'| = 1$

$$\vec{E} = \frac{-5nC}{4\pi\epsilon_0} \left(\frac{-\vec{a}_x}{1} \right)$$

so

$$\vec{E}_{\text{tot}} = \frac{5nC}{4\pi\epsilon_0} \left[\left(\frac{1}{3^{3/2}} + 1 \right) \vec{a}_x + \left(\frac{2}{2^{3/2}} - \frac{1}{3^{3/2}} \right) \vec{a}_y + \left(\frac{2}{2^{3/2}} - \frac{1}{3^{3/2}} \right) \vec{a}_z \right]$$

$$= 45 \text{ V}\cdot\text{m} \left(1.19 \vec{a}_x + 0.515 \vec{a}_y + 0.515 \vec{a}_z \right)$$

$$= 53.6 \vec{a}_x + 23.1 \vec{a}_y + 23.1 \vec{a}_z \quad \frac{\text{V}}{\text{m}}$$

(1a)

The potential is also found through superposition

10nC

$$V_1 = \frac{10nC}{4\pi\epsilon_0} \frac{1}{\sqrt{2}}$$

-5nC

$$V_2 = \frac{-5nC}{4\pi\epsilon_0} \frac{1}{\sqrt{3}}$$

-5nC

$$V_3 = \frac{-5nC}{4\pi\epsilon_0} \frac{1}{1}$$

So

$$V = \frac{5nC}{4\pi\epsilon_0} \left[\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{3}} - 1 \right]$$

so

$$V = \frac{5nC}{4\pi\epsilon_0} (-0.163)$$

$$= -7.33 \text{ V}$$

2. a) $\vec{E}(r, \theta, \varphi) = E_r(r) \vec{a}_r$

②

b) $\rho = \rho_0 (ar - r^2)$

$$\epsilon_0 \oint_S \vec{E} \cdot d\vec{S} = Q_{\text{encl.}}$$

use a spherical surface of radius r for S

$$\Rightarrow \epsilon_0 4\pi r^2 E_r = Q_{\text{encl.}}$$

outside the sphere

$$Q_{\text{encl.}} = \int_0^{2\pi} \int_0^{\pi} \int_0^a \rho_0 (ar - r^2) r^2 \sin\theta dr d\theta d\varphi$$

$$\Rightarrow Q_{\text{encl.}} = 4\pi \rho_0 \int_0^a ar^3 - r^4 dr$$

$$= 4\pi \rho_0 \left(\frac{a}{4} r^4 - \frac{1}{5} r^5 \right) \Big|_0^a$$

$$= 4\pi \rho_0 a^5 \left(\frac{1}{4} - \frac{1}{5} \right) = \frac{4\pi \rho_0 a^5}{20} = \frac{\pi \rho_0 a^5}{5}$$

so

$$E_r = \frac{\rho_0 a^5}{20 r^2 \epsilon_0}$$

(3)

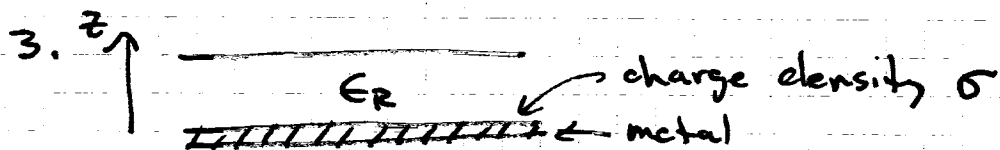
Inside the sphere

$$Q_{\text{encl.}} = \int_0^r \int_0^\pi \int_0^{2\pi} \rho_0 (ar' - r'^2) r'^2 \sin\theta \, dr' d\theta d\phi$$

$$= 4\pi\rho_0 \left(\frac{a}{4} r'^4 - \frac{1}{5} r'^5 \right) \Big|_0^r$$

$$= 4\pi\rho_0 \left(\frac{a}{4} r^4 - \frac{1}{5} r^5 \right)$$

$$\Rightarrow E_r = \frac{4\pi\rho_0 \left(\frac{a}{4} r^4 - \frac{1}{5} r^5 \right)}{4\pi\epsilon_0 r^2} = \frac{\rho_0}{\epsilon_0} \left(\frac{a}{4} r^2 - \frac{1}{5} r^3 \right)$$

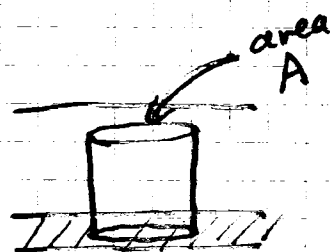


Use Gauss's Law to find \vec{D} which depends only on the free charge σ .

By symmetry $\vec{D} = D_z(z) \vec{a}_z$

$$\oint_S \vec{D} \cdot d\vec{S} = Q_{\text{encl.}}^{\text{free}}$$

choose for S



only contribution to the surface integral comes from the top plate of S

$$\Rightarrow AD_z = A\sigma$$

$$\Rightarrow D_z = \sigma$$

so $\vec{D} = \sigma \vec{a}_z$ This is the displacement field every where above the metal

Inside the dielectric

$$\vec{E} = \frac{\vec{D}}{\epsilon_0 \epsilon_r} = \frac{\sigma}{\epsilon_0 \epsilon_r} \vec{a}_z$$

Outside the dielectric

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\sigma}{\epsilon_0} \vec{a}_z$$

The discontinuity in \vec{E} is produced by a surface layer of bound charge at each surface of the dielectric.

since $Q_{\text{total}} = Q_{\text{free}} + Q_{\text{bound}}$

for the surface S

$$Q_{\text{enc}}^{\text{bound}} = \epsilon_0 \left(\oint_S \vec{E} \cdot d\vec{s} - \oint_S \vec{D} \cdot d\vec{s} \right)$$

$$Q_{\text{bound}} = \frac{\sigma}{\epsilon_r} A - \sigma A = \sigma A \left(\frac{1}{\epsilon_r} - 1 \right)$$

So the bound charge density on the surface of the dielectric nearest the metal is

$$\sigma_{\text{bound}} = \sigma \left(\frac{1}{\epsilon_r} - 1 \right)$$

a corresponding σ_{bound} is on the other surface but of opposite sign.

4. a) The half plane will intercept $1/4$ of the total flux.

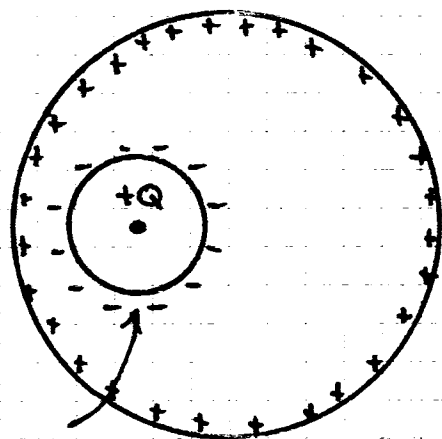
Since the total flux of a point charge is just Q the flux through the $1/2$ plane is

$$\frac{100 \mu\text{C}}{4} = 25 \mu\text{C}$$

air cap.	dielectric cap.	
$\vec{D} = \sigma \vec{a}_z$	$\vec{D} = \sigma \vec{a}_z$	same
$\vec{E} = \frac{\sigma}{\epsilon_0} \vec{a}_z$	$\vec{E} = \frac{\sigma}{\epsilon_0 \epsilon_r} \vec{a}_z$	air cap greater
$V = \frac{\sigma}{\epsilon_0} d$	$V = \frac{\sigma}{\epsilon_0 \epsilon_r} d$	air cap greater

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5. a)



uniform + charge
on metal's surface
total charge +Q

uniform -
charge on inner
surface of cavity.
total charge -Q

Inside Cavity: \vec{E} is just that of the point charge

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \vec{a}_r$$

where the origin is at Q

Inside metal $\vec{E} = 0$

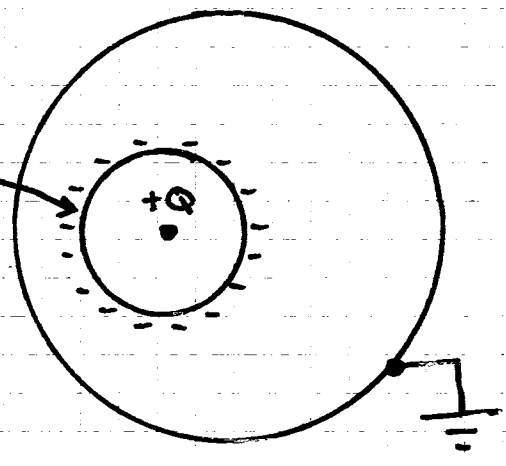
Outside metal

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \vec{a}_r$$

But now the origin is at the center of the metal sphere.

b)

Uniform -
charge on inner
surface of cavity
total charge -Q



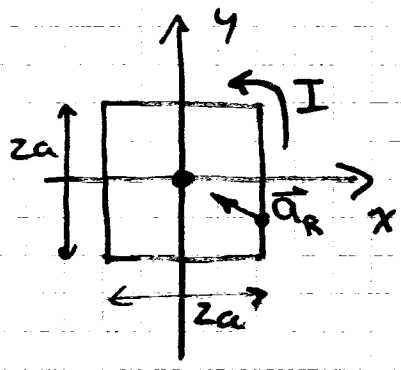
No other charge in metal

Inside cavity $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \vec{a}_r$ origin at Q

Outside metal $\vec{E} = 0$

Inside metal $\vec{E} = 0$

6.



$$\vec{H} = \oint \frac{I d\vec{l} \times \vec{a}_R}{4\pi R^2}$$

Consider the segment at $x = +a$
the magnetic field from this segment

is

$$\vec{H}_i = \int I \frac{d\vec{l} \times \vec{a}_R}{4\pi R^2}$$

$$d\vec{l} = \vec{a}_y dy$$

$$\vec{a}_R = \frac{-a\vec{a}_x - y\vec{a}_y}{(a^2 + y^2)^{1/2}}$$

so $d\vec{l} \times \vec{a}_R = \frac{a\vec{a}_z}{(a^2 + y^2)^{1/2}} dy$

and $R = (a^2 + y^2)^{1/2}$

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So

$$\begin{aligned}\vec{H}_1 &= \frac{I}{4\pi} \int_{-a}^a \frac{a \, dy}{(a^2 + y^2)^{3/2}} \vec{a}_z \\ &= \frac{Ia}{4\pi} \left[\frac{y}{a^2 (a^2 + y^2)^{1/2}} \right]_{-a}^a \vec{a}_z \\ &= \frac{Ia}{4\pi} \frac{2a \vec{a}_z}{a^2 (a^2 + a^2)^{1/2}} = \frac{2I}{\sqrt{2} a 4\pi} \vec{a}_z\end{aligned}$$

A moment's thought should convince you that the other three segments produce the same field vector at the center of the loop.

So

$$\vec{H} = \frac{2I}{\sqrt{2} a \pi} \vec{a}_z$$